

3月28日更新题目:

一个最简真分数 $\frac{M}{7}$, 化成小数后, 如果从小数点后第一位起连续若干位的数字之和等于 2004, 求 M 的值.

3月27日更新题目答案:

请在下面的方框内填入加号或减号, 以使得下面的关系式成立:

$$0 < 1 \square \frac{1}{2} \square \frac{1}{3} \square \cdots \square \frac{1}{19} < \frac{1}{97}$$

【解析】: 由于前面几个数较大, 可以考虑先抵消, 有 $1 - \frac{1}{2} - \frac{1}{3} - \frac{1}{6} = 0$, $\frac{1}{4} - \frac{1}{8} - \frac{1}{16} = \frac{1}{16}$, $\frac{1}{7} - \frac{1}{14} = \frac{1}{14}$, $\frac{1}{9} - \frac{1}{18} = \frac{1}{18}$, $\frac{1}{5} - \frac{1}{10} = \frac{1}{10}$: 则

$$0 < 1 \square \frac{1}{2} \square \frac{1}{3} \square \frac{1}{4} \square \frac{1}{5} \square \frac{1}{6} \square \frac{1}{7} \square \frac{1}{8} \square \frac{1}{9} \square \frac{1}{10} \square \frac{1}{11} \square \frac{1}{12} \square \frac{1}{13} \square \frac{1}{14} \square \frac{1}{15} \square \frac{1}{16} \square \frac{1}{17} \square \frac{1}{18} \square \frac{1}{19} < \frac{1}{97}$$

\Rightarrow

$$0 < 1 \square \frac{1}{2} \square \frac{1}{3} \square \frac{1}{6} \square \frac{1}{4} \square \frac{1}{8} \square \frac{1}{16} \square \frac{1}{5} \square \frac{1}{10} \square \frac{1}{7} \square \frac{1}{14} \square \frac{1}{9} \square \frac{1}{18} \square \frac{1}{11} \square \frac{1}{12} \square \frac{1}{13} \square \frac{1}{15} \square \frac{1}{17} \square \frac{1}{19} < \frac{1}{97}$$

$$\Rightarrow 0 < \left(1 - \frac{1}{2} - \frac{1}{3} - \frac{1}{6}\right) + \left(\frac{1}{4} - \frac{1}{8} - \frac{1}{16}\right) \square \left(\frac{1}{5} - \frac{1}{10}\right) \square \left(\frac{1}{7} - \frac{1}{14}\right) \square \left(\frac{1}{9} - \frac{1}{18}\right) \square \frac{1}{11} \square \frac{1}{12} \square \frac{1}{13} \square \frac{1}{15} \square \frac{1}{17} \square \frac{1}{19} < \frac{1}{97}$$

$$\Rightarrow 0 < \frac{1}{16} \square \frac{1}{10} \square \frac{1}{14} \square \frac{1}{18} \square \frac{1}{11} \square \frac{1}{12} \square \frac{1}{13} \square \frac{1}{15} \square \frac{1}{17} \square \frac{1}{19} < \frac{1}{97}$$

$$\Rightarrow 0 < \left(\frac{1}{10} - \frac{1}{11}\right) \square \left(\frac{1}{12} - \frac{1}{13}\right) \square \left(\frac{1}{14} - \frac{1}{15}\right) \square \left(\frac{1}{16} - \frac{1}{17}\right) \square \left(\frac{1}{18} - \frac{1}{19}\right) < \frac{1}{97}$$

$$\text{又 } \frac{1}{n} - \frac{1}{n+1} = \frac{n+1-n}{n(n+1)} = \frac{1}{n(n+1)},$$

$$\text{则 } 0 < \left(\frac{1}{10} - \frac{1}{11}\right) \square \left(\frac{1}{12} - \frac{1}{13}\right) \square \left(\frac{1}{14} - \frac{1}{15}\right) \square \left(\frac{1}{16} - \frac{1}{17}\right) \square \left(\frac{1}{18} - \frac{1}{19}\right) < \frac{1}{97}$$

$$\Rightarrow 0 < \frac{1}{10 \times 11} \square \frac{1}{12 \times 13} \square \frac{1}{14 \times 15} \square \frac{1}{16 \times 17} \square \frac{1}{18 \times 19} < \frac{1}{97}$$

$$\text{又 } 0 < \left(\frac{1}{10 \times 11} - \frac{1}{12 \times 13}\right) + \left(\frac{1}{14 \times 15} - \frac{1}{16 \times 17}\right) + \frac{1}{18 \times 19}$$

$$\text{且 } \frac{1}{10 \times 11} - \left(\frac{1}{12 \times 13} - \frac{1}{14 \times 15}\right) - \left(\frac{1}{16 \times 17} - \frac{1}{18 \times 19}\right) < \frac{1}{10 \times 11} < \frac{1}{97}$$

所以

$$0 < \frac{1}{10 \times 11} - \frac{1}{12 \times 13} + \frac{1}{14 \times 15} - \frac{1}{16 \times 17} + \frac{1}{18 \times 19} < \frac{1}{97}$$

$$\Rightarrow 0 < \left(\frac{1}{10} - \frac{1}{11}\right) - \left(\frac{1}{12} - \frac{1}{13}\right) + \left(\frac{1}{14} - \frac{1}{15}\right) - \left(\frac{1}{16} - \frac{1}{17}\right) + \left(\frac{1}{18} - \frac{1}{19}\right) < \frac{1}{97}$$

$$\Rightarrow 0 < \left(1 - \frac{1}{2} - \frac{1}{3} - \frac{1}{6}\right) + \left(\frac{1}{5} - \frac{1}{10}\right) - \frac{1}{11} - \frac{1}{12} + \frac{1}{13} + \left(\frac{1}{7} - \frac{1}{14}\right) - \frac{1}{15} - \left(\frac{1}{4} - \frac{1}{8} - \frac{1}{16}\right) + \frac{1}{17} + \left(\frac{1}{9} - \frac{1}{18}\right) - \frac{1}{19} < \frac{1}{97}$$

\Rightarrow

$$1 - \frac{1}{2} - \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \frac{1}{9} - \frac{1}{10} - \frac{1}{11} - \frac{1}{12} + \frac{1}{13} - \frac{1}{14} - \frac{1}{15} + \frac{1}{16} + \frac{1}{17} - \frac{1}{18} - \frac{1}{19} < \frac{1}{97}$$