

## 公式法习题答案

1、计算:  $1+4+7+10+13+\cdots+100$ 

$$\begin{aligned}\text{原式} &= \frac{(1+100) \times 34}{2} \\ &= 1717\end{aligned}$$

2、计算:  $11+13+15+\cdots+99$ 

$$\begin{aligned}\text{原式} &= 50^2 - 5^2 \\ &= 2475\end{aligned}$$

3、计算:  $\frac{363636}{363363} \times \frac{636636}{636363}$ 

$$\begin{aligned}\text{原式} &= \frac{36 \times 10101}{363 \times 1001} \times \frac{636 \times 1001}{63 \times 10101} \\ &= \frac{4}{121} \times \frac{212}{7} \\ &= \frac{848}{847}\end{aligned}$$

4、计算:  $\left(1 - \frac{1}{2^2}\right) \times \left(1 - \frac{1}{3^2}\right) \times \cdots \times \left(1 - \frac{1}{99^2}\right)$ 

$$\begin{aligned}\text{原式} &= \left(1 - \frac{1}{2}\right) \times \left(1 + \frac{1}{2}\right) \times \left(1 - \frac{1}{3}\right) \times \left(1 + \frac{1}{3}\right) \times \cdots \times \left(1 - \frac{1}{99}\right) \times \left(1 + \frac{1}{99}\right) \\ &= \frac{1}{2} \times \frac{3}{2} \times \frac{2}{3} \times \frac{4}{3} \times \cdots \times \frac{98}{99} \times \frac{100}{99} \\ &= \frac{50}{99}\end{aligned}$$

5、计算:  $\frac{1}{2} + \frac{1}{2+4} + \frac{1}{2+4+6} + \frac{1}{2+4+6+8} + \cdots + \frac{1}{2+4+6+\cdots+20}$ 

$$\begin{aligned}\text{原式} &= \frac{1}{2} + \frac{1}{\frac{(2+4) \times 2}{2}} + \frac{1}{\frac{(2+6) \times 3}{2}} + \frac{1}{\frac{(2+8) \times 4}{2}} + \cdots + \frac{1}{\frac{(2+20) \times 10}{2}} \\ &= \frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \frac{1}{4 \times 5} + \cdots + \frac{1}{10 \times 11} \\ &= 1 - \frac{1}{11} \\ &= \frac{10}{11}\end{aligned}$$

6、计算:  $\frac{2^2+1}{2^2-1} + \frac{4^2+1}{4^2-1} + \cdots + \frac{18^2+1}{18^2-1} + \frac{20^2+1}{20^2-1}$

$$\begin{aligned} \text{原式} &= \frac{2^2-1+2}{2^2-1} + \frac{4^2-1+2}{4^2-1} + \cdots + \frac{18^2-1+2}{18^2-1} + \frac{20^2-1+2}{20^2-1} \\ &= 1 + \frac{2}{2^2-1} + 1 + \frac{2}{4^2-1} + \cdots + 1 + \frac{2}{18^2-1} + 1 + \frac{2}{20^2-1} \\ &= 10 + 2 \times \left( \frac{1}{2^2-1} + \frac{1}{4^2-1} + \cdots + \frac{1}{18^2-1} + \frac{1}{20^2-1} \right) \\ &= 10 + 2 \times \left( \frac{1}{1 \times 3} + \frac{1}{3 \times 5} + \cdots + \frac{1}{17 \times 19} + \frac{1}{19 \times 21} \right) \\ &= 10 + \frac{20}{21} \end{aligned}$$

7、计算:  $\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \cdots + \frac{99}{100!}$  (最后结果可以用阶乘表示)

$$\begin{aligned} \text{原式} &= \frac{2-1}{2!} + \frac{3-1}{3!} + \frac{4-1}{4!} + \cdots + \frac{100-1}{100!} \\ &= \frac{1}{1!} - \frac{1}{2!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{3!} - \frac{1}{4!} + \cdots + \frac{1}{99!} - \frac{1}{100!} \\ &= 1 - \frac{1}{100!} \end{aligned}$$

8、计算:  $\frac{3! \times 1}{3} + \frac{4! \times 2}{3^2} + \frac{5! \times 3}{3^3} + \cdots + \frac{102! \times 100}{3^{100}}$  (结果可以用阶乘和乘方表示)

$$\begin{aligned} \text{原式} &= \frac{3! \times 1}{3} + \frac{4! \times 2}{3^2} + \frac{5! \times 3}{3^3} + \cdots + \frac{102! \times 100}{3^{100}} \\ &= \frac{3! \times (4-3)}{3} + \frac{4! \times (5-3)}{3^2} + \frac{5! \times (6-3)}{3^3} + \cdots + \frac{102! \times (103-3)}{3^{100}} \\ &= \frac{4!}{3} - \frac{3!}{1} + \frac{5!}{3^2} - \frac{4!}{3} + \frac{6!}{3^3} - \frac{5!}{3^2} + \cdots + \frac{103!}{3^{100}} - \frac{102!}{3^{99}} \\ &= \frac{103!}{3^{100}} - 3! \end{aligned}$$

9、计算:  $1^2 + 3^2 + 5^2 + \cdots + 19^2$

$$\begin{aligned} \text{原式} &= (1^2 + 2^2 + 3^2 + \cdots + 19^2) - (2^2 + 4^2 + \cdots + 18^2) \\ &= \frac{1}{6} \times 19 \times 20 \times 39 - 4 \times (1^2 + 2^2 + \cdots + 9^2) \\ &= 2470 - \frac{1}{6} \times 9 \times 10 \times 19 \\ &= 2470 - 285 \\ &= 2185 \end{aligned}$$

10、计算:  $\frac{1+2^3+3^3+\cdots+2006^3}{1+2+3+\cdots+2006}$

$$\begin{aligned} \text{原式} &= \frac{(1+2+3+\cdots+2006)^2}{1+2+3+\cdots+2006} \\ &= 1+2+3+\cdots+2006 \\ &= \frac{1}{2} \times 2006 \times (2006+1) \\ &= 2013021 \end{aligned}$$