

## 裂项习题答案

1、计算:  $\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \frac{1}{4 \times 5} + \frac{1}{5 \times 6} + \frac{1}{6 \times 7} + \frac{1}{7 \times 8} + \frac{1}{8 \times 9} + \frac{1}{9 \times 10}$

$$\begin{aligned}\text{原式} &= \frac{1}{1} - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \frac{1}{4} - \frac{1}{5} + \frac{1}{5} - \frac{1}{6} + \frac{1}{6} - \frac{1}{7} + \frac{1}{7} - \frac{1}{8} + \frac{1}{8} - \frac{1}{9} + \frac{1}{9} - \frac{1}{10} \\ &= 1 - \frac{1}{10} \\ &= \frac{9}{10}\end{aligned}$$

2、计算:  $\frac{2}{1 \times 3} + \frac{2}{3 \times 5} + \frac{2}{5 \times 7} + \dots + \frac{2}{97 \times 99}$

$$\begin{aligned}\text{原式} &= \frac{1}{1} - \frac{1}{3} + \frac{1}{3} - \frac{1}{5} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{97} - \frac{1}{99} \\ &= 1 - \frac{1}{99} \\ &= \frac{98}{99}\end{aligned}$$

3、计算:  $\frac{1}{2 \times 4} + \frac{1}{4 \times 6} + \frac{1}{6 \times 8} + \dots + \frac{1}{98 \times 100}$

$$\begin{aligned}\text{原式} &= \frac{1}{2} \times \left( \frac{1}{1} - \frac{1}{4} \right) + \frac{1}{2} \times \left( \frac{1}{4} - \frac{1}{6} \right) + \frac{1}{2} \times \left( \frac{1}{6} - \frac{1}{8} \right) + \dots + \frac{1}{2} \times \left( \frac{1}{98} - \frac{1}{100} \right) \\ &= \frac{1}{2} \times \left( 1 - \frac{1}{99} \right) \\ &= \frac{49}{99}\end{aligned}$$

4、计算:  $\frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \frac{1}{42} + \frac{1}{56} + \frac{1}{72} + \frac{1}{90}$

$$\begin{aligned}\text{原式} &= \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \frac{1}{4 \times 5} + \frac{1}{5 \times 6} + \frac{1}{6 \times 7} + \frac{1}{7 \times 8} + \frac{1}{8 \times 9} + \frac{1}{9 \times 10} \\ &= \frac{1}{2} - \frac{1}{10} \\ &= \frac{2}{5}\end{aligned}$$

5、计算:  $\frac{1}{4} + \frac{1}{28} + \frac{1}{70} + \frac{1}{130} + \dots + \frac{1}{9700}$

$$\begin{aligned}\text{原式} &= \frac{1}{1 \times 4} + \frac{1}{4 \times 7} + \frac{1}{7 \times 10} + \frac{1}{10 \times 13} + \dots + \frac{1}{97 \times 100} \\ &= \frac{1}{3} \times \left( \frac{1}{1} - \frac{1}{4} + \frac{1}{4} - \frac{1}{7} + \frac{1}{7} - \frac{1}{10} + \frac{1}{10} - \frac{1}{13} + \dots + \frac{1}{97} - \frac{1}{100} \right) \\ &= \frac{1}{3} \times \frac{99}{100} \\ &= \frac{33}{100}\end{aligned}$$

6、计算:  $\frac{2}{1 \times 2 \times 3} + \frac{2}{2 \times 3 \times 4} + \frac{2}{3 \times 4 \times 5} + \dots + \frac{2}{98 \times 99 \times 100}$

$$\begin{aligned}\text{原式} &= \frac{1}{1 \times 2} - \frac{1}{2 \times 3} + \frac{1}{2 \times 3} - \frac{1}{3 \times 4} + \frac{1}{3 \times 4} - \frac{1}{4 \times 5} + \dots + \frac{1}{98 \times 99} - \frac{1}{99 \times 100} \\ &= \frac{1}{2} - \frac{1}{99 \times 100} \\ &= \frac{4949}{9900}\end{aligned}$$

7、计算:  $\frac{5+6}{5 \times 6} - \frac{6+7}{6 \times 7} + \frac{7+8}{7 \times 8} - \frac{8+9}{8 \times 9} + \frac{9+10}{9 \times 10}$

$$\begin{aligned}\text{原式} &= \frac{6}{5 \times 6} + \frac{5}{5 \times 6} - \frac{7}{6 \times 7} - \frac{6}{6 \times 7} + \frac{8}{7 \times 8} + \frac{7}{7 \times 8} - \frac{9}{8 \times 9} - \frac{8}{8 \times 9} + \frac{10}{9 \times 10} + \frac{9}{9 \times 10} \\ &= \frac{1}{5} + \frac{1}{6} - \frac{1}{6} - \frac{1}{7} + \frac{1}{7} + \frac{1}{8} - \frac{1}{8} - \frac{1}{9} + \frac{1}{9} + \frac{1}{10} \\ &= \frac{1}{5} + \frac{1}{10} \\ &= \frac{3}{10}\end{aligned}$$

8、计算:  $\frac{3}{2} - \frac{5}{6} + \frac{7}{12} - \frac{9}{20} + \frac{11}{30} - \frac{13}{42} + \frac{15}{56} - \frac{17}{72} + \frac{19}{90}$

$$\begin{aligned}\text{原式} &= \frac{1+2}{1 \times 2} - \frac{2+3}{2 \times 3} + \frac{3+4}{3 \times 4} - \frac{4+5}{4 \times 5} + \frac{5+6}{5 \times 6} - \frac{6+7}{6 \times 7} + \frac{7+8}{7 \times 8} - \frac{8+9}{8 \times 9} + \frac{9+10}{9 \times 10} \\ &= \frac{1}{1} + \frac{1}{2} - \frac{1}{2} - \frac{1}{3} + \frac{1}{3} + \frac{1}{4} - \frac{1}{4} - \frac{1}{5} + \frac{1}{5} + \frac{1}{6} - \frac{1}{6} - \frac{1}{7} + \frac{1}{7} + \frac{1}{8} - \frac{1}{8} - \frac{1}{9} + \frac{1}{9} + \frac{1}{10} \\ &= 1 + \frac{1}{10} \\ &= \frac{11}{10}\end{aligned}$$

9、计算:  $1\frac{1}{2}+3\frac{1}{6}+5\frac{1}{12}+7\frac{1}{20}+9\frac{1}{30}+11\frac{1}{42}+13\frac{1}{56}+15\frac{1}{72}+17\frac{1}{90}$

$$\begin{aligned}\text{原式} &= (1+3+5+7+9+11+13+15+17) + \left( \frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \frac{1}{4 \times 5} + \frac{1}{5 \times 6} + \frac{1}{6 \times 7} + \frac{1}{7 \times 8} + \frac{1}{8 \times 9} + \frac{1}{9 \times 10} \right) \\ &= \frac{(1+17) \times 9}{2} + \left( \frac{1}{1} - \frac{1}{10} \right) \\ &= 81\frac{9}{10}\end{aligned}$$

10、计算:  $\frac{3}{2} + \frac{11}{6} + \frac{23}{12} + \frac{39}{20} + \dots + \frac{759}{380} + \frac{839}{420}$

$$\begin{aligned}\text{原式} &= 2 - \frac{1}{2} + 2 - \frac{1}{6} + 2 - \frac{1}{12} + 2 - \frac{1}{20} + \dots + 2 - \frac{1}{380} + 2 - \frac{1}{420} \\ &= 2 \times 20 - \left( \frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \frac{1}{4 \times 5} + \dots + \frac{1}{19 \times 20} + \frac{1}{20 \times 21} \right) \\ &= 40 - \left( 1 - \frac{1}{21} \right) \\ &= 39\frac{1}{21}\end{aligned}$$

11、计算:  $\frac{1^2+2^2}{1 \times 2} + \frac{2^2+3^2}{2 \times 3} + \dots + \frac{18^2+19^2}{18 \times 19} + \frac{19^2+20^2}{19 \times 20}$

$$\begin{aligned}\text{原式} &= \frac{2}{1} + \frac{1}{2} + \frac{3}{2} + \frac{2}{3} + \dots + \frac{19}{18} + \frac{18}{19} + \frac{20}{19} + \frac{19}{20} \\ &= 2 \times 19 + \frac{19}{20} \\ &= 38\frac{19}{20}\end{aligned}$$