

裂项知识点与习题

知识点

一、裂差

$$1、\frac{1}{1 \times 2} = \frac{1}{1} - \frac{1}{2}, \frac{1}{2 \times 3} = \frac{1}{2} - \frac{1}{3}, \frac{1}{3 \times 4} = \frac{1}{3} - \frac{1}{4}, \frac{1}{4 \times 5} = \frac{1}{4} - \frac{1}{5}, \dots, \frac{1}{n \times (n+1)} = \frac{1}{n} - \frac{1}{n+1}$$

$$2、\frac{1}{1 \times 3} = \frac{1}{2} \times \left(\frac{1}{1} - \frac{1}{3} \right), \frac{1}{2 \times 4} = \frac{1}{2} \times \left(\frac{1}{2} - \frac{1}{4} \right), \frac{1}{3 \times 5} = \frac{1}{2} \times \left(\frac{1}{3} - \frac{1}{5} \right), \dots, \frac{1}{n \times (n+2)} = \frac{1}{2} \times \left(\frac{1}{n} - \frac{1}{n+2} \right)$$

$$3、\frac{1}{1 \times 4} = \frac{1}{3} \times \left(\frac{1}{1} - \frac{1}{4} \right), \frac{1}{2 \times 5} = \frac{1}{3} \times \left(\frac{1}{2} - \frac{1}{5} \right), \frac{1}{3 \times 6} = \frac{1}{3} \times \left(\frac{1}{3} - \frac{1}{6} \right), \dots, \frac{1}{n \times (n+3)} = \frac{1}{3} \times \left(\frac{1}{n} - \frac{1}{n+3} \right)$$

$$\frac{1}{n \times (n+d)} = \frac{1}{d} \times \left(\frac{1}{n} - \frac{1}{n+d} \right)$$

$$4、\frac{1}{1 \times 2 \times 3} = \frac{1}{2} \times \left(\frac{1}{1 \times 2} - \frac{1}{2 \times 3} \right), \frac{1}{2 \times 3 \times 4} = \frac{1}{2} \times \left(\frac{1}{2 \times 3} - \frac{1}{3 \times 4} \right), \frac{1}{3 \times 4 \times 5} = \frac{1}{2} \times \left(\frac{1}{3 \times 4} - \frac{1}{4 \times 5} \right)$$

$$\frac{1}{n \times (n+1) \times (n+2)} = \frac{1}{2} \times \left[\frac{1}{n \times (n+1)} - \frac{1}{(n+1) \times (n+2)} \right]$$

$$5、\frac{1}{1 \times 2 \times 3 \times 4} = \frac{1}{3} \times \left(\frac{1}{1 \times 2 \times 3} - \frac{1}{2 \times 3 \times 4} \right), \frac{1}{2 \times 3 \times 4 \times 5} = \frac{1}{3} \times \left(\frac{1}{2 \times 3 \times 4} - \frac{1}{3 \times 4 \times 5} \right), \frac{1}{3 \times 4 \times 5 \times 6} = \frac{1}{3} \times \left(\frac{1}{3 \times 4 \times 5} - \frac{1}{4 \times 5 \times 6} \right)$$

$$\frac{1}{n \times (n+1) \times (n+2) \times (n+3)} = \frac{1}{3} \times \left[\frac{1}{n \times (n+1) \times (n+2)} - \frac{1}{(n+1) \times (n+2) \times (n+3)} \right]$$

二、裂和

$$1、\frac{a+b}{a \times b} = \frac{a}{a \times b} + \frac{b}{a \times b} = \frac{1}{b} + \frac{1}{a}$$

$$2、\frac{a^2+b^2}{a \times b} = \frac{a^2}{a \times b} + \frac{b^2}{a \times b} = \frac{a}{b} + \frac{b}{a}$$

习题

1、计算: $\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \frac{1}{4 \times 5} + \frac{1}{5 \times 6} + \frac{1}{6 \times 7} + \frac{1}{7 \times 8} + \frac{1}{8 \times 9} + \frac{1}{9 \times 10}$

2、计算: $\frac{2}{1 \times 3} + \frac{2}{3 \times 5} + \frac{2}{5 \times 7} + \dots + \frac{2}{97 \times 99}$

3、计算: $\frac{1}{2 \times 4} + \frac{1}{4 \times 6} + \frac{1}{6 \times 8} + \dots + \frac{1}{98 \times 100}$

4、计算: $\frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \frac{1}{42} + \frac{1}{56} + \frac{1}{72} + \frac{1}{90}$

5、计算: $\frac{1}{4} + \frac{1}{28} + \frac{1}{70} + \frac{1}{130} + \dots + \frac{1}{9700}$

6、计算: $\frac{2}{1 \times 2 \times 3} + \frac{2}{2 \times 3 \times 4} + \frac{2}{3 \times 4 \times 5} + \dots + \frac{2}{98 \times 99 \times 100}$

7、计算: $\frac{5+6}{5 \times 6} - \frac{6+7}{6 \times 7} + \frac{7+8}{7 \times 8} - \frac{8+9}{8 \times 9} + \frac{9+10}{9 \times 10}$

8、计算: $\frac{3}{2} - \frac{5}{6} + \frac{7}{12} - \frac{9}{20} + \frac{11}{30} - \frac{13}{42} + \frac{15}{56} - \frac{17}{72} + \frac{19}{90}$

9、计算: $1\frac{1}{2} + 3\frac{1}{6} + 5\frac{1}{12} + 7\frac{1}{20} + 9\frac{1}{30} + 11\frac{1}{42} + 13\frac{1}{56} + 15\frac{1}{72} + 17\frac{1}{90}$

10、计算: $\frac{3}{2} + \frac{11}{6} + \frac{23}{12} + \frac{39}{20} + \dots + \frac{759}{380} + \frac{839}{420}$

11、计算: $\frac{1^2 + 2^2}{1 \times 2} + \frac{2^2 + 3^2}{2 \times 3} + \dots + \frac{18^2 + 19^2}{18 \times 19} + \frac{19^2 + 20^2}{19 \times 20}$