

通项归纳习题答案

1、计算: $1 + \frac{2}{1 \times (1+2)} + \frac{3}{(1+2) \times (1+2+3)} + \frac{4}{(1+2+3) \times (1+2+3+4)} + \cdots + \frac{10}{(1+2+\cdots+9) \times (1+2+\cdots+9+10)}$

$$\begin{aligned} a_n &= \frac{n}{(1+2+3+\cdots+n-1) \times (1+2+3+\cdots+n)} \\ &= \frac{n}{\frac{(1+n-1) \times (n-1)}{2} \times \frac{(1+n) \times n}{2}} \\ &= \frac{4}{(n-1) \times n \times (n+1)} \end{aligned}$$

$$\begin{aligned} \text{原式} &= 1 + \frac{2}{1 \times \frac{(1+2) \times 2}{2}} + \frac{3}{\frac{(1+2) \times 2}{2} \times \frac{(1+3) \times 3}{2}} + \frac{4}{\frac{(1+3) \times 3}{2} \times \frac{(1+4) \times 4}{2}} + \cdots + \frac{10}{\frac{(1+9) \times 9}{2} \times \frac{(1+10) \times 10}{2}} \\ &= 1 + \frac{4}{1 \times 2 \times 3} + \frac{4}{2 \times 3 \times 4} + \frac{4}{3 \times 4 \times 5} + \cdots + \frac{4}{9 \times 10 \times 11} \\ &= 1 + 2 \times \left(\frac{1}{1 \times 2} - \frac{1}{2 \times 3} + \frac{1}{2 \times 3} - \frac{1}{3 \times 4} + \frac{1}{3 \times 4} - \frac{1}{4 \times 5} + \cdots + \frac{1}{9 \times 10} - \frac{1}{10 \times 11} \right) \\ &= 1 \frac{54}{55} \end{aligned}$$

2、计算: $\frac{2 \times 3}{1 \times 4} + \frac{5 \times 6}{4 \times 7} + \frac{8 \times 9}{7 \times 10} + \cdots + \frac{98 \times 99}{97 \times 100}$

$$\begin{aligned} a_n &= \frac{(3n-1) \times 3n}{(3n-2) \times (3n+1)} \\ &= \frac{9n^2 - 3n}{9n^2 - 3n - 2} \\ &= \frac{9n^2 - 3n - 2 + 2}{9n^2 - 3n - 2} \\ &= 1 + \frac{2}{9n^2 - 3n - 2} \\ &= 1 + \frac{1}{(3n-2) \times (3n+1)} \end{aligned}$$

$$\begin{aligned} \text{原式} &= 1 + \frac{2}{1 \times 4} + 1 + \frac{2}{4 \times 7} + 1 + \frac{2}{7 \times 10} + \cdots + 1 + \frac{2}{97 \times 100} \\ &= 33 + 2 \times \frac{1}{3} \times \left(1 - \frac{1}{4} + \frac{1}{4} - \frac{1}{7} + \frac{1}{7} - \frac{1}{10} + \cdots + \frac{1}{97} - \frac{1}{100} \right) \\ &= 33 \frac{33}{50} \end{aligned}$$

3、计算: $\frac{4}{1 \times 2 \times 3} + \frac{5}{2 \times 3 \times 4} + \frac{6}{3 \times 4 \times 5} + \cdots + \frac{11}{8 \times 9 \times 10}$

$$a_n = \frac{n+3}{n \times (n+1) \times (n+2)}$$

$$= \frac{1}{(n+1) \times (n+2)} + \frac{3}{(n+1) \times (n+2)}$$

$$\begin{aligned} \text{原式} &= \frac{1}{2 \times 3} + \frac{3}{1 \times 2 \times 3} + \frac{1}{3 \times 4} + \frac{3}{2 \times 3 \times 4} + \frac{1}{4 \times 5} + \frac{3}{3 \times 4 \times 5} + \cdots + \frac{1}{9 \times 10} + \frac{3}{8 \times 9 \times 10} \\ &= \left(\frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \frac{1}{4 \times 5} + \cdots + \frac{1}{9 \times 10} \right) + \left(\frac{3}{1 \times 2 \times 3} + \frac{3}{2 \times 3 \times 4} + \frac{3}{3 \times 4 \times 5} + \cdots + \frac{3}{8 \times 9 \times 10} \right) \\ &= \frac{1}{2} - \frac{1}{10} + 3 \times \frac{1}{2} \times \left(\frac{1}{1 \times 2} - \frac{1}{9 \times 10} \right) \\ &= 1 \frac{2}{15} \end{aligned}$$

4、计算: $\left(1 + \frac{1}{1 \times 3}\right) \times \left(1 + \frac{1}{2 \times 4}\right) \times \left(1 + \frac{1}{3 \times 5}\right) \times \cdots \times \left(1 + \frac{1}{2007 \times 2009}\right)$

$$a_n = 1 + \frac{1}{n \times (n+2)}$$

$$= \frac{n^2 + 2n + 1}{n \times (n+2)}$$

$$= \frac{(n+1)^2}{n \times (n+2)}$$

$$\begin{aligned} \text{原式} &= \frac{2^2}{1 \times 3} \times \frac{3^2}{2 \times 4} \times \frac{4^2}{3 \times 5} \times \cdots \times \frac{2008^2}{2007 \times 2009} \\ &= \frac{2}{1} \times \frac{2}{3} \times \frac{3}{2} \times \frac{3}{4} \times \frac{4}{3} \times \frac{3}{5} \times \cdots \times \frac{2008}{2007} \times \frac{2008}{2009} \\ &= 2 \times \frac{2008}{2009} \\ &= \frac{4016}{2009} \end{aligned}$$

5、计算: $\frac{3}{1 \times 2 \times 3} + \frac{5}{2 \times 3 \times 5} + \frac{7}{3 \times 4 \times 5} + \cdots + \frac{37}{18 \times 19 \times 20}$

$$a_n = \frac{2 \times n + 1}{n \times (n+1) \times (n+2)}$$

$$= \frac{2}{(n+1) \times (n+2)} + \frac{1}{n \times (n+1) \times (n+2)}$$

$$\begin{aligned} \text{原式} &= \frac{2}{2 \times 3} + \frac{1}{1 \times 2 \times 3} + \frac{2}{3 \times 4} + \frac{1}{2 \times 3 \times 4} + \frac{2}{4 \times 5} + \frac{1}{3 \times 4 \times 5} + \cdots + \frac{2}{19 \times 20} + \frac{1}{18 \times 19 \times 20} \\ &= \left(\frac{2}{2 \times 3} + \frac{2}{3 \times 4} + \frac{2}{4 \times 5} + \cdots + \frac{2}{19 \times 20} \right) + \left(\frac{1}{1 \times 2 \times 3} + \frac{1}{2 \times 3 \times 4} + \frac{1}{3 \times 4 \times 5} + \cdots + \frac{1}{18 \times 19 \times 20} \right) \\ &= 2 \times \left(\frac{1}{2} - \frac{1}{20} \right) + \frac{1}{2} \times \left(\frac{1}{1 \times 2} - \frac{1}{19 \times 20} \right) \\ &= \frac{813}{760} \end{aligned}$$