

2018 年苏州市初中毕业暨升学考试

数学试题参考答案

一、选择题：（每小题 3 分，共 30 分）

1. C 2. C 3. B 4. D 5. B
6. C 7. B 8. D 9. B 10. A

二、填空题：（每小题 3 分，共 24 分）

11. a^3 12. 8 13. -2 14. 12
15. 80 16. $\frac{2}{3}$ 17. $\frac{4}{5}$ 18. $2\sqrt{3}$

三、解答题：（共 76 分）

19. 解：原式 $= \frac{1}{2} + 3 - \frac{1}{2}$
 $= 3.$

20. 解：由 $3x \geq x + 2$ ，解得 $x \geq 1$ ，
由 $x + 4 < 2(2x - 1)$ ，解得 $x > 2$ ，
 $\therefore$ 不等式组的解集是 $x > 2$.

21. 证明： $\because AB \parallel DE$ ， $\therefore \angle A = \angle D$.
 $\because AF = DC$ ， $\therefore AC = DF$.

在 $\triangle ABC$ 和 $\triangle DEF$ 中， $\begin{cases} AB = DE, \\ \angle A = \angle D, \\ AC = DF, \end{cases}$

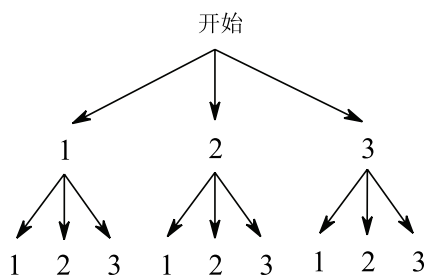
$\therefore \triangle ABC \cong \triangle DEF$ (SAS).

$\therefore \angle ACB = \angle DFE$,

$\therefore BC \parallel EF$.

22. 解：(1) $\frac{2}{3}$;

(2) 用“树状图”或利用表格列出所有可能的结果：



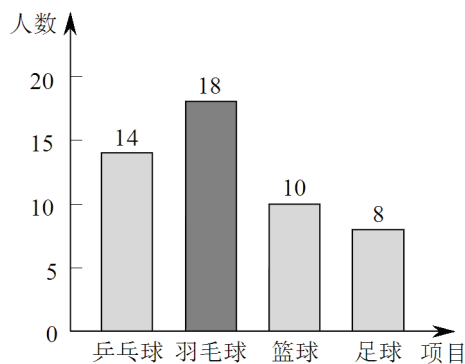
第1次 和 第2次	1	2	3
1	2	3	4
2	3	4	5
3	4	5	6

$\therefore P$ (两个数字之和是 3 的倍数) $= \frac{3}{9} = \frac{1}{3}$.

23. 解: (1) $\frac{14}{28\%}=50$,

答: 参加这次调查的学生人数为 50 人.

补全条形统计图如图所示:



(2) $\frac{10}{50} \times 360^\circ = 72^\circ$,

答: 扇形统计图中“篮球”项目所对应扇形的圆心角度数为 72° .

(3) $600 \times \frac{8}{50} = 96$,

答: 估计该校选择“足球”项目的学生有 96 人.

24. 解: (1) 设每台 A 型电脑的价格为 x 元, 每台 B 型打印机的价格为 y 元.

根据题意得:
$$\begin{cases} x + 2y = 5900, \\ 2x + 2y = 9400. \end{cases}$$

解这个方程组, 得
$$\begin{cases} x = 3500, \\ y = 1200. \end{cases}$$

答: 每台 A 型电脑的价格为 3500 元, 每台 B 型打印机的价格为 1200 元.

(2) 设学校购买 n 台 B 型打印机, 则购买 A 型电脑为 $(n-1)$ 台.

根据题意得: $3500(n-1) + 1200n \leq 20000$,

解这个不等式, 得 $n \leq 5$.

答: 该学校至多能购买 5 台 B 型打印机.

25. 解: (1) 由 $x^2 - 4 = 0$ 解得 $x_1 = 2$, $x_2 = -2$.

$\because$ 点 A 位于点 B 的左侧, $\therefore A(-2, 0)$.

$\because$ 直线 $y = x + m$ 经过点 A, $\therefore -2 + m = 0$,

$\therefore m = 2$, $\therefore D(0, 2)$.

$\therefore AD = \sqrt{OA^2 + OD^2} = 2\sqrt{2}$.

(2) 解法一：设新抛物线对应的函数表达式为 $y = x^2 + bx + 2$,

$$\therefore y = x^2 + bx + 2 = \left(x + \frac{b}{2}\right)^2 + 2 - \frac{b^2}{4}, \therefore C' \left(-\frac{b}{2}, 2 - \frac{b^2}{4}\right).$$

$\therefore$ 直线 CC' 平行于直线 AD , 并且经过点 $C(0, -4)$,

$\therefore$ 直线 CC' 的函数表达式为 $y = x - 4$.

$$\therefore 2 - \frac{b^2}{4} = -\frac{b}{2} - 4, \text{ 整理得 } b^2 - 2b - 24 = 0, \text{ 解得 } b_1 = -4, b_2 = 6.$$

$\therefore$ 新抛物线对应的函数表达式为 $y = x^2 - 4x + 2$ 或 $y = x^2 + 6x + 2$.

解法二： $\therefore$ 直线 CC' 平行于直线 AD , 并且经过点 $C(0, -4)$,

$\therefore$ 直线 CC' 的函数表达式为 $y = x - 4$.

$\therefore$ 新抛物线的顶点 C' 在直线 $y = x - 4$ 上, $\therefore$ 设顶点 C' 的坐标为 $(n, n - 4)$,

$\therefore$ 新抛物线对应的函数表达式为 $y = (x - n)^2 + n - 4$.

$\therefore$ 新抛物线经过点 $D(0, 2)$,

$$\therefore n^2 + n - 4 = 2, \text{ 解得 } n_1 = -3, n_2 = 2.$$

$\therefore$ 新抛物线对应的函数表达式为 $y = (x + 3)^2 - 7$ 或 $y = (x - 2)^2 - 2$.

26. 证明: (1) 连接 AC .

$\therefore CD$ 为 $\odot O$ 的切线, $\therefore OC \perp CD$.

又 $\therefore AD \perp CD$, $\therefore \angle DCO = \angle D = 90^\circ$,

$\therefore AD \parallel OC$, $\therefore \angle DAC = \angle ACO$.

又 $\therefore OC = OA$, $\therefore \angle CAO = \angle ACO$,

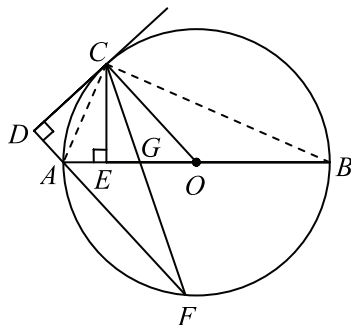
$\therefore \angle DAC = \angle CAO$.

又 $\therefore CE \perp AB$, $\therefore \angle CEA = 90^\circ$.

$$\text{在 } \triangle CDA \text{ 和 } \triangle CEA \text{ 中, } \begin{cases} \angle D = \angle CEA, \\ \angle DAC = \angle EAC, \\ AC = AC, \end{cases}$$

$\therefore \triangle CDA \cong \triangle CEA$ (AAS),

$\therefore CD = CE$.



(2) 证法一：连接 BC .

$$\because \triangle CDA \cong \triangle CEA, \therefore \angle DCA = \angle ECA,$$

$$\because CE \perp AG, AE = EG, \therefore CA = CG.$$

$$\therefore \angle ECA = \angle ECG.$$

$$\because AB \text{ 是 } \odot O \text{ 的直径}, \therefore \angle ACB = 90^\circ.$$

$$\text{又 } \because CE \perp AB,$$

$$\therefore \angle ACE = \angle B.$$

$$\text{又 } \because \angle B = \angle F,$$

$$\therefore \angle F = \angle ACE = \angle DCA = \angle ECG.$$

$$\text{又 } \because \angle D = 90^\circ,$$

$$\therefore \angle DCF + \angle F = 90^\circ,$$

$$\therefore \angle F = \angle DCA = \angle ACE = \angle ECG = 22.5^\circ,$$

$$\therefore \angle AOC = 2\angle F = 45^\circ.$$

$$\therefore \triangle CEO \text{ 是等腰直角三角形.}$$

证法二：设 $\angle F = x^\circ$,

$$\text{则 } \angle AOC = 2\angle F = 2x^\circ.$$

$$\because AD \parallel OC, \therefore \angle OAF = \angle AOC = 2x^\circ,$$

$$\therefore \angle CGA = \angle OAF + \angle F = 3x^\circ,$$

$$\because CE \perp AG, AE = EG, \therefore CA = CG, \therefore \angle EAC = \angle CGA,$$

$$\therefore \angle DAC = \angle EAC = \angle CGA = 3x^\circ.$$

$$\text{又 } \because \angle DAC + \angle EAC + \angle OAF = 180^\circ,$$

$$\therefore 3x^\circ + 3x^\circ + 2x^\circ = 180^\circ,$$

$$\therefore x = 22.5,$$

$$\therefore \angle AOC = 2x^\circ = 45^\circ.$$

$$\therefore \triangle CEO \text{ 是等腰直角三角形.}$$

27. 解：问题 1：(1) $\frac{3}{16}$;

(2) 解法一： $\because AB=4, AD=m, \therefore BD=4-m$.

$$\text{又 } \because DE \parallel BC, \therefore \frac{CE}{EA} = \frac{BD}{DA} = \frac{4-m}{m}, \therefore \frac{S_{\triangle DEC}}{S_{\triangle ADE}} = \frac{4-m}{m}.$$

$$\text{又 } \because DE \parallel BC, \therefore \triangle ADE \sim \triangle ABC, \therefore \frac{S_{\triangle ADE}}{S_{\triangle ABC}} = \left(\frac{m}{4}\right)^2 = \frac{m^2}{16}.$$

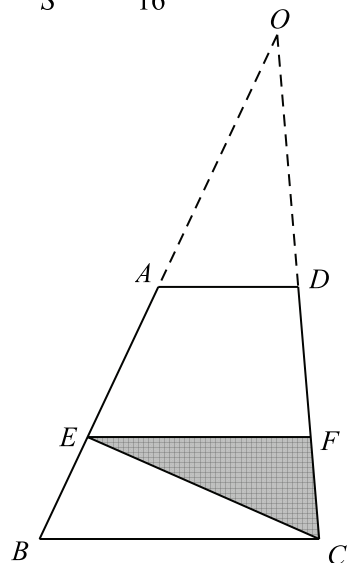
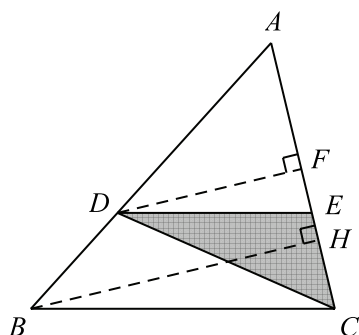
$$\therefore \frac{S_{\triangle DEC}}{S_{\triangle ABC}} = \frac{S_{\triangle DEC}}{S_{\triangle ADE}} \times \frac{S_{\triangle ADE}}{S_{\triangle ABC}} = \frac{4-m}{m} \times \frac{m^2}{16} = \frac{-m^2 + 4m}{16}, \text{ 即 } \frac{S'}{S} = \frac{-m^2 + 4m}{16}.$$

解法二：过点 B 作 $BH \perp AC$ ，垂足为 H ，过点 D 作 $DF \perp AC$ ，垂足为 F 。

$$\text{则 } DF \parallel BH, \therefore \triangle ADF \sim \triangle ABH, \therefore \frac{DF}{BH} = \frac{AD}{AB} = \frac{m}{4}.$$

$$\because DE \parallel BC, \therefore \frac{CE}{CA} = \frac{BD}{BA} = \frac{4-m}{4}.$$

$$\therefore \frac{S_{\triangle DEC}}{S_{\triangle ABC}} = \frac{\frac{1}{2}CE \cdot DF}{\frac{1}{2}CA \cdot BH} = \frac{4-m}{4} \times \frac{m}{4} = \frac{-m^2 + 4m}{16}, \text{ 即 } \frac{S'}{S} = \frac{-m^2 + 4m}{16}.$$



问题 2：解法一：分别延长 BA ， CD ，相交于点 O 。

$$\because AD \parallel BC, \therefore \triangle OAD \sim \triangle OBC, \therefore \frac{OA}{OB} = \frac{AD}{BC} = \frac{1}{2}.$$

$$\therefore OA = AB = 4, \therefore OB = 8.$$

$$\because AE = n, \therefore OE = 4 + n.$$

$$\because EF \parallel BC,$$

$$\text{由问题 1 的解法可知, } \frac{S_{\triangle CEF}}{S_{\triangle OBC}} = \frac{S_{\triangle CEF}}{S_{\triangle OEF}} \times \frac{S_{\triangle OEF}}{S_{\triangle OBC}} = \frac{4-n}{4+n} \times \left(\frac{4+n}{8}\right)^2 = \frac{16-n^2}{64}.$$

$$\therefore \frac{S_{\triangle OAD}}{S_{\triangle OBC}} = \left(\frac{OA}{OB}\right)^2 = \frac{1}{4}, \therefore \frac{S_{ABCD}}{S_{\triangle OBC}} = \frac{3}{4}.$$

$$\therefore \frac{S_{\triangle CEF}}{S_{ABCD}} = \frac{S_{\triangle CEF}}{\frac{3}{4}S_{\triangle OBC}} = \frac{4}{3} \times \frac{16-n^2}{64} = \frac{16-n^2}{48}, \text{ 即 } \frac{S'}{S} = \frac{16-n^2}{48}.$$

解法二：连接 AC 交 EF 于 M .

$$\because AD \parallel BC, \text{ 且 } AD = \frac{1}{2}BC, \therefore \frac{S_{\triangle ADC}}{S_{\triangle ABC}} = \frac{1}{2}.$$

$$\therefore S_{\triangle ADC} = \frac{1}{3}S, \quad S_{\triangle ABC} = \frac{2}{3}S.$$

$$\text{由问题 1 的结论可知, } \frac{S_{\triangle EMC}}{S_{\triangle ABC}} = \frac{-n^2 + 4n}{16},$$

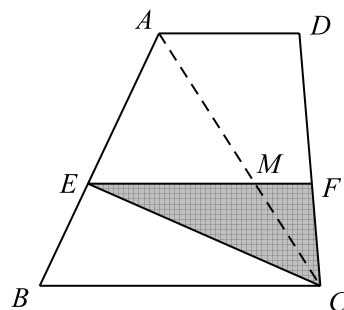
$$\therefore S_{\triangle EMC} = \frac{-n^2 + 4n}{16} \times \frac{2}{3}S = \frac{-n^2 + 4n}{24} \times S.$$

$$\because MF \parallel AD,$$

$$\therefore \triangle CFM \sim \triangle CDA,$$

$$\therefore \frac{S_{\triangle CFM}}{S_{\triangle CDA}} = \frac{S_{\triangle CFM}}{\frac{1}{3}S} = 3 \times \frac{S_{\triangle CFM}}{S} = \left(\frac{4-n}{4}\right)^2,$$

$$\therefore S_{\triangle CFM} = \frac{(4-n)^2}{48} \times S.$$



$$\therefore S_{\triangle EFC} = S_{\triangle EMC} + S_{\triangle CFM} = \frac{-n^2 + 4n}{24} \times S + \frac{(4-n)^2}{48} \times S = \frac{16-n^2}{48} \times S,$$

$$\therefore \frac{S'}{S} = \frac{16-n^2}{48}.$$

28. 解：(1) 设线段 MN 所在直线的函数表达式为 $y = kx + b$.

$\because M, N$ 两点的坐标分别为 $(30, 230), (100, 300)$,

$$\therefore \begin{cases} 30k + b = 230, \\ 100k + b = 300. \end{cases}$$

$$\text{解这个方程组, 得 } \begin{cases} k = 1, \\ b = 200. \end{cases}$$

$\therefore$ 线段 MN 所在直线的函数表达式为 $y = x + 200$.

(2) ①第一种情况：考虑 $FE = FG$ 是否成立.

连接 EC .

$$\because AE = x, AD = 100, GA = x + 200,$$

$$\therefore ED = GD = x + 100.$$

又 $\because CD \perp EG$, $\therefore CE = CG$,

$\therefore \angle CGE = \angle CEG$,

$\therefore \angle FEG > \angle CGE$.

$\therefore FE \neq FG$.

②第二种情况：考虑 $FG = EG$ 是否成立.

$\because$ 四边形 $ABCD$ 是正方形, $\therefore BC \parallel EG$, $\therefore \triangle FBC \sim \triangle FEG$.

假设 $FG = EG$ 成立, 则 $FC = BC$ 亦成立.

$\therefore FC = BC = 100$.

$\because AE = x$, $GA = x + 200$,

$\therefore FG = EG = AE + GA = 2x + 200$,

$\therefore CG = FG - FC = 2x + 200 - 100 = 2x + 100$.

在 $\text{Rt}\triangle CDG$ 中, $CD = 100$, $GD = x + 100$, $CG = 2x + 100$,

$$\therefore 100^2 + (x + 100)^2 = (2x + 100)^2,$$

解这个方程, 得 $x_1 = -100$, $x_2 = \frac{100}{3}$.

$\because x > 0$, $\therefore x = \frac{100}{3}$.

③第三种情况：考虑 $EF = EG$ 是否成立.

与②同理, 假设 $EF = EG$ 成立, 则 $FB = BC$ 亦成立.

$\therefore BE = EF - FB = 2x + 200 - 100 = 2x + 100$.

在 $\text{Rt}\triangle ABE$ 中, $AE = x$, $AB = 100$, $BE = 2x + 100$,

$$\therefore 100^2 + x^2 = (2x + 100)^2,$$

解这个方程, 得 $x_1 = 0$, $x_2 = -\frac{400}{3}$ (不合题意, 均舍去).

综上所述, 当 $x = \frac{100}{3}$ 时, $\triangle EFG$ 是一个等腰三角形.

