

## 第五届“学习之星”评比活动试卷

### 七年级数学

#### 一、选择题(本题共 10 小题, 每小题 3 分, 共 30 分)

1. A;      2. B;      3. C;      4. B;      5. D;      6. C;      7. D;  
8. C;      9. B;      10. C.

#### 二、填空题(本题共 6 小题, 每小题 3 分, 共 18 分)

11.  $5\sqrt{2}$ ;    12.  $\frac{8}{5}$ ;    13.  $<$ ;    14.  $30^\circ$ ;    15.  $(1, -3)$ ;    16.  $(1-\sqrt{2}, 0)$ .

#### 三、解答题(本题共 39 分, 17、18、19 小题各 9 分, 20 题 12 分)

17. (1) 
$$\begin{cases} x-3y=4 & \text{①} \\ 2x+y=1 & \text{②} \end{cases}$$

解: ① $\times$ 2 得  $2x-6y=8$  ③.....1 分

②-③得  $7y=-7$

解得  $y=-1$ .....2 分

将  $y=-1$  代入①得  $x=1$ .....3 分

所以此方程组的解为  $\begin{cases} x=1 \\ y=-1 \end{cases}$ .....4 分

(2) 
$$\begin{cases} 5x+2y=4 & \text{①} \\ 3x+4y=-6 & \text{②} \end{cases}$$

解: ① $\times$ 2 得  $10x+4y=8$  ③.....5 分

②-③得  $-7x=-14$ .....6 分

解得  $x=2$ .....7 分

将  $x=2$  代入①得  $y=-3$ .....8 分

所以此方程组的解为  $\begin{cases} x=2 \\ y=-3 \end{cases}$ .....9 分

18. 解: (1) 根据题意得:  $a+6+2a-9=0$ .....3 分

解得  $a=1$ .....4 分

(2) 因为  $a=1$ , 所以  $x^2-16=0$ .....5 分

解得  $x=4$  或  $x=-4$ .....9 分

19.解:  $\because OF \perp CD$

$\therefore \angle DOF = 90^\circ$  .....1 分

$\because \angle AOD + \angle DOF + \angle BOF = 180^\circ, \angle AOD = 50^\circ$

$\therefore \angle BOF = 180^\circ - 90^\circ - 50^\circ = 40^\circ$  .....4 分

$\because \angle AOD = \angle BOC$

$\therefore \angle BOC = 50^\circ$  .....6 分

$\because OP$  平分  $\angle BOC$

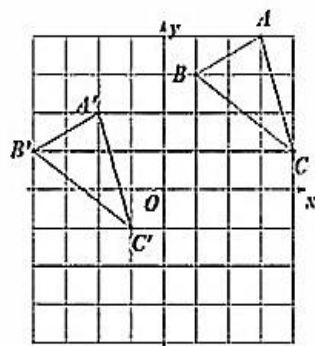
$\therefore \angle BOP = \frac{1}{2} \angle BOC = 25^\circ$  .....7 分

$\therefore \angle DOP = \angle DOF + \angle BOF + \angle BOP = 90^\circ + 40^\circ + 25^\circ = 155^\circ$  .....9 分

20.解: (1) 结论 1 分, 图形 5 分, 共 6 分

(2)  $B'(-4, 1), C'(-1, -1)$  .....10 分

(3)  $P'(a-5, b-2)$  .....12 分



#### 四. 解答题(本题共 28 分, 21、22 题各 9 分, 23 题 10 分)

21. 解: 将  $a=2, b=1$  代入

得  $\begin{cases} 4+m-1=2 \\ 2n+1=1 \end{cases}$  .....2 分

解得  $\begin{cases} m=-1 \\ n=0 \end{cases}$  .....4 分

$\therefore m+n=-1$  .....5 分

$\therefore (m+n)^{2008} = (-1)^{2008} = 1$  .....7 分

$\therefore \pm\sqrt{1} = \pm 1$  .....9 分

22.解: 设 A 饮料生产了  $x$  瓶, B 饮料生产了  $y$  瓶 .....1 分

由题意得:  $\begin{cases} x+y=100 \\ 2x+3y=270 \end{cases}$  .....6 分

解得:  $\begin{cases} x=30 \\ y=70 \end{cases}$ , .....8分

答: A 饮料生产了 30 瓶, B 饮料生产了 70 瓶. ....9分

23.证明:  $\because \angle 1 + \angle 2 = 180^\circ$ ,  $\angle 2 = \angle 4$

$\therefore \angle 1 + \angle 4 = 180^\circ$  .....2分

$\therefore DF \parallel AB$ .....4分

$\therefore \angle B = \angle FDH$ .....6分

$\because \angle 3 = \angle B$

$\therefore \angle 3 = \angle FDH$ .....8分

$\therefore EF \parallel BC$ .....10分

## 五. 解答题(本题共 35 分, 24 题 11 分, 25、26 题各 12 分)

24. (1) 过点 A、B、C 分别做 x 轴和 y 轴的平行线交于点 M、N、D.....1分

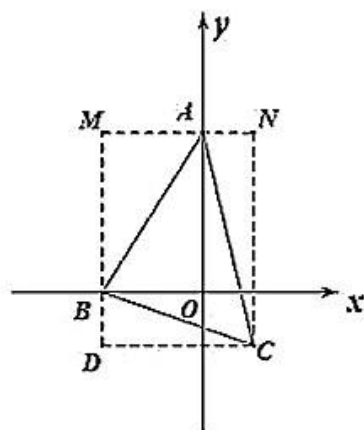
$\because A(0, 3)$ 、 $B(-2, 0)$ 、 $C(1, -1)$

$\therefore AM=2$ ,  $BM=3$ ,  $AN=1$ ,  $NC=4$ ,  $CD=3$ ,  $BD=1$ ,  $MN=3$ .....2分

$\therefore S_{\text{长方形}MNCD} = MN \cdot NC = 12$ ,  $S_{\triangle ABM} = \frac{1}{2} AM \cdot MB = 3$ ,

$S_{\triangle ACN} = \frac{1}{2} AN \cdot NC = 2$ ,  $S_{\triangle BDC} = \frac{1}{2} BD \cdot DC = \frac{3}{2}$  .....4分

$\therefore S_{\triangle ABC} = S_{\text{长方形}MNCD} - S_{\triangle ABM} - S_{\triangle ACN} - S_{\triangle BDC} = 12 - 3 - 2 - \frac{3}{2} = \frac{11}{2}$  .....5分



$$(2) \because S_{\triangle ABC} = \frac{11}{2}$$

$$\therefore \frac{1}{2} S_{\triangle ABC} = \frac{11}{4}$$

$$\therefore S_{\triangle PAB} = \frac{1}{2} BP \cdot OA = \frac{3}{2} BP = \frac{11}{4} \dots\dots\dots 6 \text{ 分}$$

$$\therefore BP = \frac{11}{6} \dots\dots\dots 7 \text{ 分}$$

$$\because B(-2, 0)$$

$$\therefore OP = OB + BP = \frac{23}{6} \text{ 或 } OP = OB - BP = \frac{1}{6} \dots\dots\dots 9 \text{ 分}$$

$$\therefore P(-\frac{23}{6}, 0) \text{ 或 } P(-\frac{1}{6}, 0) \dots\dots\dots 11 \text{ 分}$$

25.(1)证明: 过点B作BE//AC.....1分

$$\because BE \parallel AC$$

$$\therefore \angle A = \angle EBD, \angle C = \angle CBE \dots\dots\dots 2 \text{ 分}$$

$$\because \angle CBD = \angle CBE + \angle EBD$$

$$\therefore \angle CBD = \angle A + \angle C \dots\dots\dots 3 \text{ 分}$$

$$(2) \text{解: } \angle A = 2\angle F \dots\dots\dots 4 \text{ 分}$$

【法1】如图1

过点F作FH//AC交AD于点H,过点F作FM//AD,

$$\text{设 } \angle A = \alpha, \angle F = \beta$$

$$\because CF \text{ 平分 } \angle ACB$$

$$\therefore \angle ACF = \angle BCF = x$$

$$\because FH \parallel AC$$

$$\therefore \angle ACF = \angle CFH = x, \angle A = \angle AHF = \alpha \dots\dots\dots 5 \text{ 分}$$

$$\because FM \parallel AD$$

$$\therefore \angle MFH = \angle AHF = \alpha, \angle MFB + \angle FBD = 180^\circ \dots\dots\dots 6 \text{ 分}$$

由上个问题可知  $\angle CBD = \angle A + \angle C = \alpha + 2x$

$$\because \angle CBF = \angle DBF$$

$$\therefore \angle FBD = \frac{360^\circ - \alpha - 2x}{2} = 180^\circ - x - \frac{\alpha}{2} \dots\dots\dots 8 \text{ 分}$$

$$\therefore \angle MFB = 180^\circ - (180^\circ - x - \frac{\alpha}{2}) = x + \frac{\alpha}{2} \dots\dots\dots 9 \text{ 分}$$

$$\because \angle MFB = \angle MFH + \angle HFB = \alpha + x - \beta$$

$$\therefore x + \frac{\alpha}{2} = \alpha + x - \beta \dots\dots\dots 11 \text{ 分}$$

$$\therefore \alpha = 2\beta$$

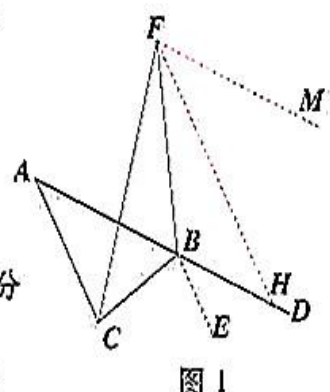


图1



$\therefore \angle A = 2\angle F$  ..... 12 分

**【法 2】** 如图 2

过点  $B$  作  $BM \parallel CF$ ,

设  $\angle A = \alpha$ ,  $\angle F = \beta$

$\because CF$  平分  $\angle ACB$

$\therefore \angle ACF = \angle BCF = x$

$\because BM \parallel CF$

$\therefore \angle F = \angle FBM = \beta$ ,  $\angle FCB + \angle CBM = 180^\circ$  ..... 5 分

由上个问题可知  $\angle CBD = \angle A + \angle C = \alpha + 2x$

$\therefore \angle CBF = \angle DBF$

$\therefore \angle FBC = \frac{360^\circ - \alpha - 2x}{2} = 180^\circ - x - \frac{\alpha}{2}$  ..... 7 分

$\because \angle MBC + \angle BCF = 180^\circ$

$\therefore x + 180^\circ - x - \frac{\alpha}{2} + \beta = 180^\circ$  ..... 11 分

$\therefore \alpha = 2\beta$

$\therefore \angle A = 2\angle F$  ..... 12 分

**【法 3】** 如图 3

过点  $B$  作  $BM \parallel CF$ , 延长  $FB$  于点  $N$

设  $\angle A = \alpha$ ,  $\angle F = \beta$

$\because CF$  平分  $\angle ACB$

$\therefore \angle ACF = \angle BCF = x$

$\because BM \parallel CF$

$\therefore \angle F = \angle MBN = \beta$ ,  $\angle FCB = \angle CBM = x$  ..... 5 分

由上个问题可知  $\angle CBD = \angle A + \angle C = \alpha + 2x$

$\therefore \angle CBF = \angle DBF$

$\therefore \angle FBC = \frac{360^\circ - \alpha - 2x}{2} = 180^\circ - x - \frac{\alpha}{2}$  ..... 7 分

$\because \angle FBC + \angle CBM + \angle MBN = 180^\circ$

$\therefore x + 180^\circ - x - \frac{\alpha}{2} + \beta = 180^\circ$  ..... 11 分

$\therefore \alpha = 2\beta$

$\therefore \angle A = 2\angle F$  ..... 12 分

**【法 4】** 如图 4

过点  $C$  作  $CN \parallel BF$ ,

设  $\angle A = \alpha$ ,  $\angle F = \beta$

$\because CF$  平分  $\angle ACB$

$\therefore \angle ACF = \angle BCF = x$

$\because CN \parallel BF$

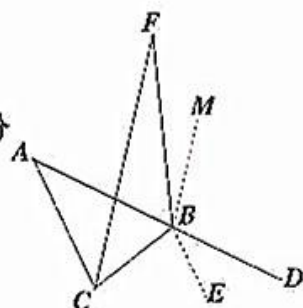


图 2

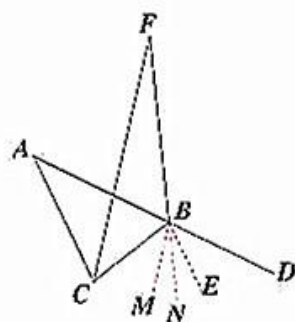


图 3

$$\therefore \angle F = \angle FCN = \beta, \angle FBC + \angle BCN = 180^\circ \dots\dots\dots 5 \text{分}$$

由上个问题可知  $\angle CBD = \angle A + \angle C = \alpha + 2x$

$$\because \angle CBF = \angle DBF$$

$$\therefore \angle FBC = \frac{360^\circ - \alpha - 2x}{2} = 180^\circ - x - \frac{\alpha}{2} \dots\dots\dots 7 \text{分}$$

$$\because \angle FBC + \angle BCN = 180^\circ$$

$$\therefore x + 180^\circ - x - \frac{\alpha}{2} + \beta = 180^\circ \dots\dots\dots 11 \text{分}$$

$$\therefore \alpha = 2\beta$$

$$\therefore \angle A = 2\angle F \dots\dots\dots 12 \text{分}$$

【法5】如图5

过点A作  $MN \parallel BF$  交FC延长线于点M, 过点A作  $AH \parallel FC$

设  $\angle A = \alpha, \angle F = \beta$

$\because CF$  平分  $\angle ACB$

$$\therefore \angle ACF = \angle BCF = x$$

$\because MN \parallel BF$

$$\therefore \angle F = \angle M = \beta, \angle FBD = \angle NAD \dots\dots\dots 5 \text{分}$$

$\because AH \parallel FC$

$$\therefore \angle M = \angle AHM = \beta, \angle HAC = \angle ACF = x \dots\dots\dots 6 \text{分}$$

由上个问题可知  $\angle CBD = \angle A + \angle C = \alpha + 2x$

$$\because \angle CBF = \angle DBF$$

$$\therefore \angle FBD = \frac{360^\circ - \alpha - 2x}{2} = 180^\circ - x - \frac{\alpha}{2} \dots\dots\dots 8 \text{分}$$

$$\because \angle NAD + \angle DAC + \angle CAM = 180^\circ$$

$$\therefore 180^\circ - x - \frac{\alpha}{2} + \alpha + x - \beta = 180^\circ \dots\dots\dots 11 \text{分}$$

$$\therefore \alpha = 2\beta$$

$$\therefore \angle A = 2\angle F \dots\dots\dots 12 \text{分}$$

$$26. (1) E(m, \frac{2}{3}m); F(0, m) \dots\dots\dots 2 \text{分}$$

$$(2) \because E(m, \frac{2}{3}m); D(m, 0)$$

$$\therefore OD = m, DE = \frac{2}{3}m$$

$\because$  正方形OABC的边长为20

$$\therefore OA = AB = 20$$

如图1, 当  $0 < m \leq 20$  时

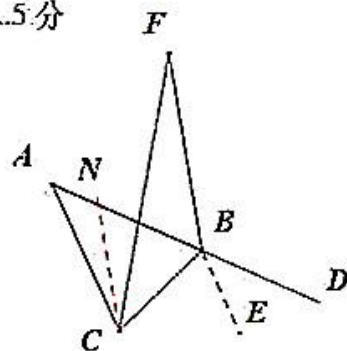


图4

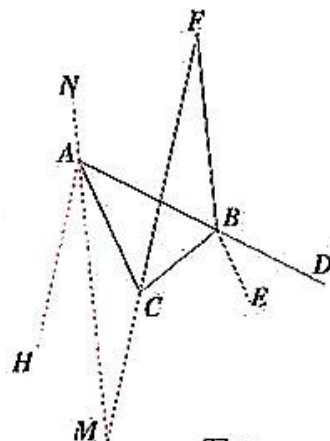


图5

$$S=OD \cdot DE=\frac{2}{3}m^2 \dots\dots\dots 4 \text{ 分}$$

如图 2, 当  $20 < m \leq 30$  时

$$S=OA \cdot AH=20 \times \frac{2}{3}m=\frac{40}{3}m \dots\dots\dots 6 \text{ 分}$$

如图 3, 当  $m > 30$  时

$$S=OA \cdot AB=20 \times 20=400 \dots\dots\dots 7 \text{ 分}$$

综上所述,  $S = \begin{cases} \frac{2}{3}m^2 & 0 < m \leq 20 \\ \frac{40}{3}m & 20 < m \leq 30 \\ 400 & m > 30 \end{cases} \dots\dots\dots 8 \text{ 分}$

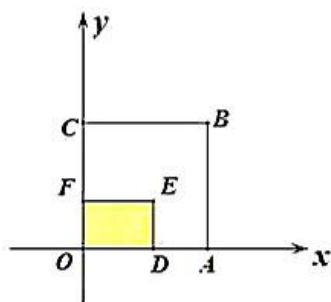


图 1

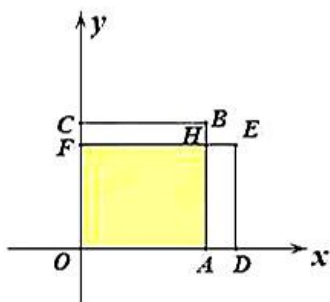


图 2

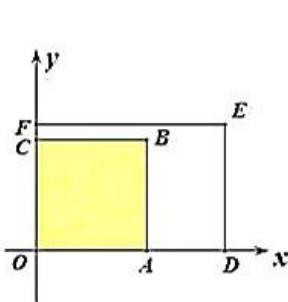


图 3

(3) 当  $\frac{2}{3}m^2=300$  时, 有  $m=\sqrt{450}$  或  $m=-\sqrt{450}$  (舍).....9 分

因为  $\sqrt{450} > 20$ , 不符合题意, 所以舍去; .....10 分

当  $\frac{40}{3}m=300$  时,  $m=\frac{45}{2}$ , 符合题意.....12 分