

## 2019-2020 深圳高级中学九年级上学期期末数学参考答案与解析

### 一. 选择题 (共 12 小题)

|   |   |   |   |   |   |   |   |   |    |    |    |
|---|---|---|---|---|---|---|---|---|----|----|----|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| D | D | A | C | D | A | C | B | A | D  | A  | B  |

12. 解: 在正方形  $ABCD$  中,  $AB = BC = AD$ ,  $\angle ABC = \angle BAD = 90^\circ$ ,

$\because E$ 、 $F$  分别为边  $AB$ ,  $BC$  的中点,

$$\therefore AE = BF = \frac{1}{2}BC,$$

$$\text{在 } \triangle ABF \text{ 和 } \triangle DAE \text{ 中, } \begin{cases} AE = BF \\ \angle ABC = \angle BAD, \\ AB = AD \end{cases}$$

$$\therefore \triangle ABF \cong \triangle DAE(SAS),$$

$$\therefore \angle BAF = \angle ADE,$$

$$\therefore \angle BAF + \angle DAF = \angle BAD = 90^\circ,$$

$$\therefore \angle ADE + \angle DAF = \angle BAD = 90^\circ,$$

$$\therefore \angle AMD = 180^\circ - (\angle ADE + \angle DAF) = 180^\circ - 90^\circ = 90^\circ,$$

$$\therefore \angle AME = 180^\circ - \angle AMD = 180^\circ - 90^\circ = 90^\circ,$$

故①正确;

$\because DE$  是  $\triangle ABD$  的中线,

$$\therefore \angle ADE \neq \angle EDB,$$

$$\therefore \angle BAF \neq \angle EDB,$$

故②错误;

设正方形  $ABCD$  的边长为  $2a$ , 则  $BF = a$ ,

$$\text{在 Rt}\triangle ABF \text{ 中, } AF = \sqrt{AB^2 + BF^2} = \sqrt{5}a,$$

$$\therefore \angle BAF = \angle MAE, \quad \angle ABC = \angle AME = 90^\circ,$$

$$\therefore \triangle AME \sim \triangle ABF,$$

$$\therefore \frac{AM}{AB} = \frac{AE}{AF}, \quad \text{即 } \frac{AM}{2a} = \frac{a}{\sqrt{5}a},$$

$$\text{解得: } AM = \frac{2\sqrt{5}}{5}a,$$

$$\therefore MF = AF - AM = \sqrt{5}a - \frac{2\sqrt{5}}{5}a = \frac{3\sqrt{5}}{5}a,$$

$$\therefore AM = \frac{2}{3}MF,$$

故③正确；

如图，过点  $M$  作  $MN \perp AB$  于  $N$ ，

$$\text{则 } \frac{MN}{BF} = \frac{AN}{AB} = \frac{AM}{AF},$$

$$\text{即 } \frac{MN}{a} = \frac{AN}{2a} = \frac{\frac{2\sqrt{5}}{5}a}{\sqrt{5}a},$$

$$\text{解得 } MN = \frac{2}{5}a, \quad AN = \frac{4}{5}a,$$

$$\therefore NB = AB - AN = 2a - \frac{4}{5}a = \frac{6}{5}a,$$

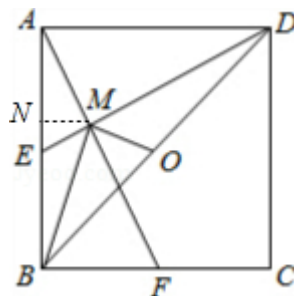
$$\text{根据勾股定理, } BM = \sqrt{BN^2 + MN^2} = \frac{2\sqrt{10}}{5}a,$$

$$\therefore ME + MF = \frac{\sqrt{5}}{5}a + \frac{3\sqrt{5}}{5}a = \frac{4\sqrt{5}}{5}a, \quad \sqrt{2}MB = \sqrt{2} \times \frac{2\sqrt{10}}{5}a = \frac{4\sqrt{5}}{5}a,$$

$$\therefore ME + MF = \sqrt{2}MB.$$

综上所述，正确的结论有①③④共 3 个。

故选：B。



## 二. 填空题（共 4 小题）

|                             |    |    |               |
|-----------------------------|----|----|---------------|
| 13                          | 14 | 15 | 16            |
| $x_1 = 0, \quad x_2 = 2020$ | -1 | 10 | $\frac{3}{2}$ |

16. 解：（1）过点  $A$  作  $AH \perp x$  轴，垂足为点  $H$ ， $AH$  交  $OC$  于点  $M$ ，如图所示。

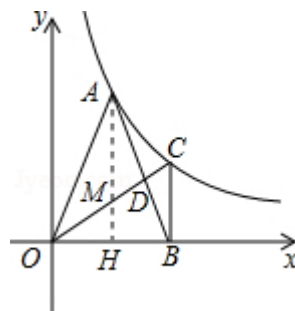
$$\because OA = AB, \quad AH \perp OB,$$

$$\therefore OH = BH = \frac{1}{2}OB,$$

$$\text{设 } OH = BH = a, \quad \text{则 } A\left(a, \frac{k}{a}\right), \quad C\left(2a, \frac{k}{2a}\right)$$

$$\therefore AH \parallel BC,$$

$$\therefore MH = \frac{1}{2}BC = \frac{k}{4a},$$



$$\therefore AM = AH - MH = \frac{k}{a} - \frac{k}{4a} = \frac{3k}{4a}.$$

$$\therefore AM \parallel BC,$$

$$\therefore \triangle ADM \sim \triangle BDC,$$

$$\therefore \frac{AD}{DB} = \frac{AM}{BC} = \frac{3}{2}.$$

### 三. 解答题 (共 7 小题)

$$\begin{aligned} 17. \text{ 解: 原式} &= \left| 1 - \frac{\sqrt{3}}{2} \right| + 2\sqrt{3} + 2 - 1 \\ &= 1 - \frac{\sqrt{3}}{2} + 2\sqrt{3} + 1 \\ &= 2 + \frac{3}{2}\sqrt{3} \end{aligned}$$

$$18. \text{ 解: (1) } (2+3) \div 10\% = 50,$$

所以参加本次比赛的选手共有 50 人,

频数直方图中“79.5~89.5”这两组的人数为  $50 \times 36\% = 18$  人,

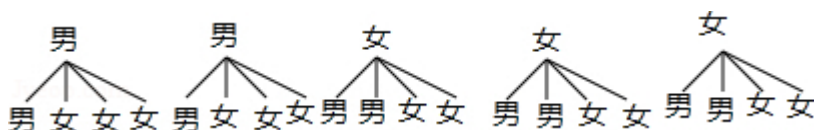
所以频数直方图中“69.5~74.5”这一组的人数为  $50 - 5 - 8 - 18 - 8 - 4 = 7$  人

“84.5~89.5”这一组的人数为  $18 - 10 = 8$  人

中位数是第 25 和第 26 位选手成绩的平均值, 即在“79.5~84.5”分数段

故答案为: 50; 79.5~84.5; 图略

(2) 画树状图为:



共有 20 种等可能的结果数, 其中恰好选中 1 男 1 女的结果数为 12,

$$\text{所以恰好选中 1 男 1 女的概率} = \frac{12}{20} = \frac{3}{5}.$$

$$19. \text{ 解: (1) 由题意得: } \angle ACB = 20^\circ + 40^\circ = 60^\circ;$$

$$(2) \text{ 由题意得, } \angle CAB = 65^\circ - 20^\circ = 45^\circ, \angle ACB = 40^\circ + 20^\circ = 60^\circ, AB = 30\sqrt{2},$$

过  $B$  作  $BE \perp AC$  于  $E$ , 如图所示:

$$\therefore \angle AEB = \angle CEB = 90^\circ,$$

在  $\text{Rt}\triangle ABE$  中,  $\because \angle ABE = 45^\circ$ ,

$\therefore \triangle ABE$  是等腰直角三角形,

$$\therefore AB = 30\sqrt{2},$$

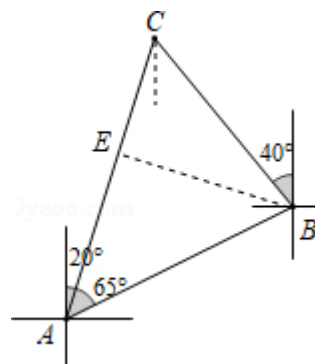
$$\therefore AE = BE = \frac{\sqrt{2}}{2} AB = 30,$$

在  $\text{Rt}\triangle CBE$  中,  $\because \angle ACB = 60^\circ$ ,  $\tan \angle ACB = \frac{BE}{CE}$ ,

$$\therefore CE = \frac{BE}{\tan 60^\circ} = \frac{30}{\sqrt{3}} = 10\sqrt{3},$$

$$\therefore AC = AE + CE = 30 + 10\sqrt{3},$$

$\therefore A, C$  两港之间的距离为  $(30 + 10\sqrt{3})\text{km}$ .



20. (1) 证明:  $\because AC \parallel BD$ ,  $AB \parallel ED$ ,

$\therefore$  四边形  $ABDE$  是平行四边形,

$\therefore AD$  平分  $\angle CAB$ ,

$$\therefore \angle CAD = \angle BAD,$$

$\because AC \parallel BD$ ,

$$\therefore \angle CAD = \angle ADB,$$

$$\therefore \angle BAD = \angle ADB,$$

$$\therefore AB = BD,$$

$\therefore$  四边形  $ABDE$  是菱形;

(2) 解:  $\because \angle ABC = 90^\circ$ ,

$$\therefore \angle GBH + \angle ABG = 90^\circ,$$

$\because AD \perp BE$ ,

$$\therefore \angle GAB + \angle ABG = 90^\circ,$$

$$\therefore \angle GAB = \angle GBH,$$

$$\therefore \cos \angle GBH = \frac{7}{8},$$

$$\therefore \cos \angle GAB = \frac{7}{8},$$

$$\therefore \frac{AB}{AH} = \frac{AG}{AB} = \frac{7}{8},$$

$\therefore$  四边形  $ABDE$  是菱形,  $BD=14$ ,

$$\therefore AB = BD = 14,$$

$$\therefore AH = 16, AG = \frac{49}{4},$$

$$\therefore GH = AH - AG = \frac{15}{4}.$$

21. 解: (1) 根据题意, 得  $y = 250 - 10(x - 45) = -10x + 700$ .

答: 每天的销售量  $y$  (件) 与销售单价  $x$  (元) 之间的函数关系式为  $y = -10x + 700$ .

(2) 销售量不低于 240 件, 得  $-10x + 700 \geq 240$ , 解得  $x \leq 46$ ,

$$\therefore 30 < x \leq 46.$$

设销售单价为  $x$  元时, 每天获取的利润是  $w$  元, 根据题意, 得

$$w = (x - 30)(-10x + 700)$$

$$= -10x^2 + 1000x - 21000$$

$$= -10(x - 50)^2 + 4000$$

$$\therefore -10 < 0,$$

所以  $x < 50$  时,  $w$  随  $x$  的增大而增大,

所以当  $x = 46$  时,  $w$  有最大值,

$$w \text{ 的最大值为 } -10(46 - 50)^2 + 4000 = 3840.$$

答: 销售单价为 46 元时, 每天获取的利润最大, 最大利润是 3840 元.

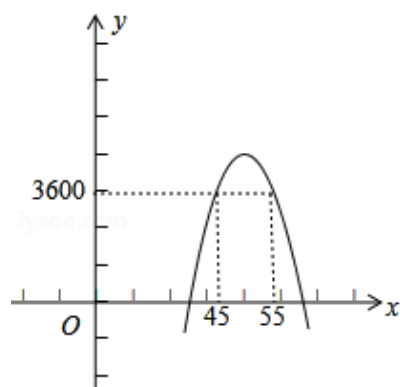
(3) 根据题意, 得

$$w - 150 = -10x^2 + 1000x - 21000 - 150 = 3600$$

$$\text{即 } -10(x - 50)^2 = -250$$

$$\text{解得 } x_1 = 55, x_2 = 45,$$

根据图象得, 当  $45 \leq x \leq 55$  时, 捐款后每天剩余利润不低于 3600 元.



22. 解：(1)  $\because DE \parallel BC$ ,

$$\therefore \frac{BD}{CE} = \frac{AD}{AE} = \frac{2}{\frac{3}{2}} = \frac{4}{3};$$

(2)  $\frac{BD}{CE}$  的值不变化, 值为  $\frac{4}{3}$ ; 理由如下:

由 (1) 得:  $DE \parallel BC$ ,

$$\therefore \triangle ADE \sim \triangle ABC,$$

$$\therefore \frac{AD}{AB} = \frac{AE}{AC},$$

由旋转的性质得:  $\angle BAD = \angle CAE$ ,

$$\therefore \triangle ABD \sim \triangle ACE,$$

$$\therefore \frac{BD}{CE} = \frac{AD}{AE} = \frac{4}{3};$$

(3) 作  $AE \perp CD$  于  $E$ ,  $DM \perp AC$  于  $M$ ,  $DN \perp BC$  于  $N$ , 如图 3 所示:

则四边形  $DMCN$  是矩形,

$$\therefore DM = CN, \quad DN = MC,$$

$$\because \angle BAC = \angle ADC = \theta, \quad \text{且 } \tan \theta = \frac{3}{4},$$

$$\therefore \frac{BC}{AC} = \frac{3}{4}, \quad \frac{AE}{DE} = \frac{3}{4},$$

$$\therefore AE = \frac{3}{5}AD = \frac{3}{5} \times 5 = 3, \quad DE = \frac{4}{3}AE = 4,$$

$$\therefore CE = CD - DE = 10 - 4 = 6,$$

$$\therefore AC = \sqrt{AE^2 + CE^2} = \sqrt{3^2 + 6^2} = 3\sqrt{5},$$

$$\therefore BC = \frac{3}{4}AC = \frac{9}{4}\sqrt{5},$$

$$\because S_{\triangle ACD} = \frac{1}{2}AC \times DM = \frac{1}{2}CD \times AE,$$

$$\therefore CN = DM = \frac{10 \times 3}{3\sqrt{5}} = 2\sqrt{5},$$

$$\therefore BN = BC + CN = \frac{9}{4}\sqrt{5} + 2\sqrt{5} = \frac{17\sqrt{5}}{4}, \quad AM = \sqrt{AD^2 - DM^2} = \sqrt{5^2 - (2\sqrt{5})^2} = \sqrt{5},$$

$$\therefore DN = MC = AM + AC = 4\sqrt{5},$$

$$\therefore BD = \sqrt{BN^2 + DN^2} = \sqrt{\left(\frac{17\sqrt{5}}{4}\right)^2 + (4\sqrt{5})^2} = \frac{5}{4}\sqrt{109}.$$

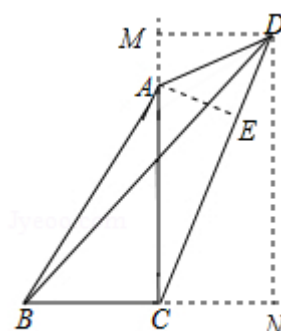


图3

23. 解: (1)  $\because y = -x^2 + (a+1)x - a$

令  $y = 0$ , 即  $-x^2 + (a+1)x - a = 0$

解得  $x_1 = a$ ,  $x_2 = 1$

由图象知:  $a < 0$

$\therefore A(a, 0)$ ,  $B(1, 0)$

$\because S_{\triangle ABC} = 6$

$\therefore \frac{1}{2}(1-a)(-a) = 6$

解得:  $a = -3$ ,  $a = 4$  (舍去)

(2) 由题意, 点  $M$  即为  $\triangle ABC$  外接圆圆心

设直线  $AC: y = kx + b$ ,

由  $A(-3, 0)$ ,  $C(0, 3)$ ,

可得  $-3k + b = 0$ , 且  $b = 3$

$\therefore k = 1$

即直线  $AC: y = x + 3$ ,

$A$ 、 $C$  的中点  $D$  坐标为  $(-\frac{3}{2}, \frac{3}{2})$

$\because A(-3, 0)$ ,  $C(0, 3)$ ,

$\therefore OA = OC$ ,

$\therefore$  线段  $AC$  的垂直平分线过原点,

$\therefore$  线段  $AC$  的垂直平分线解析式为:  $y = -x$ ,

$\because$  由  $A(-3, 0)$ ,  $B(1, 0)$ ,

$\therefore$  线段  $AB$  的垂直平分线为  $x = -1$

将  $x = -1$  代入  $y = -x$ ,

解得:  $y = 1$

$\therefore$  存在点  $M(-1, 1)$ , 使得点  $M$  到点  $A$ 、点  $B$  和点  $C$  的距离相等

(3) 作  $PM \perp x$  轴交  $x$  轴于  $M$ , 则  $S_{\triangle BAP} = \frac{1}{2} AB \cdot PM = \frac{1}{2} \times 4d$

$\because S_{\triangle PQB} = S_{\triangle PAB}$

$\therefore A$ 、 $Q$  到  $PB$  的距离相等,

$$\therefore AQ \parallel PB$$

设直线  $PB$  解析式为:  $y = x + b$

$\because$  直线经过点  $B(1,0)$

所以: 直线  $PB$  的解析式为  $y = x - 1$

$$\text{联立} \begin{cases} y = -x^2 - 2x + 3 \\ y = x - 1 \end{cases}, \text{解得: } \begin{cases} x = -4 \\ y = -5 \end{cases}$$

$\therefore$  点  $P$  坐标为  $(-4, -5)$

又  $\because \angle PAQ = \angle AQB$ ,

$\therefore \angle BPA = \angle PBQ$ ,

$\therefore AP = QB$ ,

在  $\triangle PBQ$  与  $\triangle BPA$  中,

$$\begin{cases} AP = QB \\ \angle BPA = \angle PBQ \\ PB = BP \end{cases}$$

$\therefore \triangle PBQ \cong \triangle BPA(SAS)$ ,

$\therefore PQ = AB = 4$

设  $Q(m, m+3)$

由  $PQ = 4$  得:

$$(m+4)^2 + (m+3+5)^2 = 4^2$$

解得:  $m = -4$ ,  $m = -8$  (当  $m = -8$  时,  $\angle PAQ \neq \angle AQB$ , 故应舍去)

$\therefore Q$  坐标为  $(-4, -1)$

