

青浦区 2019 学年第一学期期终学业质量调研 九年级数学试卷

参考答案及评分说明 2020.1

一、选择题:

1. A; 2. B; 3. C; 4. D; 5. A; 6. D.

二、填空题:

7. $\frac{2}{3}$; 8. $\sqrt{5}-1$; 9. $-3\vec{e}$; 10. $a>0$; 11. $>$; 12. $y=100(1+x)^2$;

13. $\frac{4\sqrt{5}}{5}$; 14. $2\sqrt{29}$; 15. 2; 16. $\frac{2\sqrt{5}}{5}$; 17. 1; 18. $\frac{5\sqrt{3}}{2}$.

三、解答题:

19. 解: 原式 $= 3 \times \frac{\sqrt{3}}{3} - \frac{1}{\frac{1}{2}} + \sqrt{8} \times \frac{\sqrt{2}}{2} + \sqrt{(1-\sqrt{3})^2}$ (8 分)

$= \sqrt{3} - 2 + 2 + \sqrt{3} - 1$ (1 分)

$= 2\sqrt{3} - 1$ (1 分)

20. 解: (1) \because 四边形 $ABCD$ 是平行四边形,

$\therefore DC \parallel AB, DC=AB$, (2 分)

$\therefore \frac{BF}{DF} = \frac{AB}{DE}$ (1 分)

$\because DE:EC=2:3, \therefore DC:DE=5:2, \therefore AB:DE=5:2$, (1 分)

$\therefore BF:DF=5:2$ (1 分)

(2) $\because BF:DF=5:2, \therefore BF = \frac{5}{7}BD$ (1 分)

$\because \overrightarrow{BD} = \overrightarrow{AD} - \overrightarrow{AB}, \therefore \overrightarrow{BD} = \vec{a} - \vec{b}$ (1 分)

$\therefore \overrightarrow{BF} = \frac{5}{7}\overrightarrow{BD} = \frac{5}{7}\vec{a} - \frac{5}{7}\vec{b}$ (1 分)

$\because \overrightarrow{AF} = \overrightarrow{AB} + \overrightarrow{BF}, \therefore \overrightarrow{AF} = \vec{b} + \frac{5}{7}\vec{a} - \frac{5}{7}\vec{b} = \frac{5}{7}\vec{a} + \frac{2}{7}\vec{b}$ (2 分)

21. 解: (1) $\because \angle ACB=90^\circ, \therefore \angle BCE+\angle GCA=90^\circ$.

$\because CG \perp BD, \therefore \angle CEB=90^\circ, \therefore \angle CBE+\angle BCE=90^\circ$,

$\therefore \angle CBE = \angle GCA$ (2 分)

又 $\because \angle DCB = \angle GAC = 90^\circ$,

$$\therefore \triangle BCD \sim \triangle CAG. \dots\dots\dots (1 \text{ 分})$$

$$\therefore \frac{CD}{AG} = \frac{BC}{CA}, \dots\dots\dots (1 \text{ 分})$$

$$\therefore \frac{1}{AG} = \frac{3}{2}, \therefore AG = \frac{2}{3}. \dots\dots\dots (1 \text{ 分})$$

$$(2) \because \angle GAC + \angle BCA = 180^\circ, \therefore GA \parallel BC. \dots\dots\dots (1 \text{ 分})$$

$$\therefore \frac{GA}{BC} = \frac{AF}{FB}. \dots\dots\dots (1 \text{ 分})$$

$$\therefore \frac{AF}{FB} = \frac{2}{9}. \dots\dots\dots (1 \text{ 分})$$

$$\therefore \frac{AF}{AB} = \frac{2}{11}. \therefore \frac{S_{\triangle AFC}}{S_{\triangle ABC}} = \frac{2}{11}. \dots\dots\dots (1 \text{ 分})$$

$$\text{又} \because S_{\triangle ABC} = \frac{1}{2} \times 2 \times 3 = 3, \therefore S_{\triangle AFC} = \frac{6}{11}. \dots\dots\dots (1 \text{ 分})$$

$$22. \text{ 解: 由题意, 得 } \angle ABD = 90^\circ, \angle D = 20^\circ, \angle ACB = 31^\circ, CD = 13. \dots\dots\dots (1 \text{ 分})$$

$$\text{在 Rt}\triangle ABD \text{ 中, } \because \tan \angle D = \frac{AB}{BD}, \therefore BD = \frac{AB}{\tan 20^\circ} = \frac{AB}{0.36}. \dots\dots\dots (3 \text{ 分})$$

$$\text{在 Rt}\triangle ABC \text{ 中, } \because \tan \angle ACB = \frac{AB}{BC}, \therefore BC = \frac{AB}{\tan 31^\circ} = \frac{AB}{0.6}. \dots\dots\dots (3 \text{ 分})$$

$$\because CD = BD - BC,$$

$$\therefore 13 = \frac{AB}{0.36} - \frac{AB}{0.6}. \dots\dots\dots (1 \text{ 分})$$

$$\text{解得 } AB \approx 11.7 \text{ 米}. \dots\dots\dots (1 \text{ 分})$$

$$\text{答: 水城门 } AB \text{ 的高约为 } 11.7 \text{ 米}. \dots\dots\dots (1 \text{ 分})$$

$$23. \text{ 证明: (1) } \because AF^2 = FG \cdot FE, \therefore \frac{AF}{FG} = \frac{FE}{AF}. \dots\dots\dots (1 \text{ 分})$$

$$\text{又} \because \angle AFG = \angle EFA, \therefore \triangle FAG \sim \triangle FEA. \dots\dots\dots (1 \text{ 分})$$

$$\therefore \angle FAG = \angle E. \dots\dots\dots (1 \text{ 分})$$

$$\because AE \parallel BC, \therefore \angle E = \angle EBC. \dots\dots\dots (1 \text{ 分})$$

$$\therefore \angle EBC = \angle FAG. \dots\dots\dots (1 \text{ 分})$$

$$\text{又} \because \angle ACD = \angle BCG, \therefore \triangle CAD \sim \triangle CBG. \dots\dots\dots (1 \text{ 分})$$

$$(2) \because \triangle CAD \sim \triangle CBG, \therefore \frac{CA}{CB} = \frac{CD}{CG}. \dots\dots\dots (1 \text{ 分})$$

又 $\because \angle DCG = \angle ACB$, $\therefore \triangle CDG \sim \triangle CAB$ (1分)

$$\therefore \frac{DG}{AB} = \frac{CG}{CB}. \dots\dots\dots (1分)$$

$$\because AE \parallel BC, \therefore \frac{AE}{CB} = \frac{AG}{GC}. \dots\dots\dots (1分)$$

$$\therefore \frac{AG}{AE} = \frac{GC}{CB}, \therefore \frac{DG}{AB} = \frac{AG}{AE}, \dots\dots\dots (1分)$$

$$\therefore DG \cdot AE = AB \cdot AG. \dots\dots\dots (1分)$$

24. 解: (1) $\because A$ 的坐标为 $(1, 0)$, 对称轴为直线 $x=2$, \therefore 点 B 的坐标为 $(3, 0)$... (1分)

将 $A(1, 0)$ 、 $B(3, 0)$ 代入 $y = x^2 + bx + c$, 得

$$\begin{cases} 1+b+c=0, \\ 9+3b+c=0. \end{cases} \quad \text{解得: } \begin{cases} b=-4, \\ c=3. \end{cases} \dots\dots\dots (2分)$$

$$\text{所以, } y = x^2 - 4x + 3.$$

$$\text{当 } x=2 \text{ 时, } y = 2^2 - 4 \times 2 + 3 = -1$$

\therefore 顶点坐标为 $(2, -1)$ (1分).

(2) 过点 P 作 $PN \perp x$ 轴, 垂足为点 N . 过点 C 作 $CM \perp PN$, 交 NP 的延长线于点 M .

$\because \angle CON = 90^\circ$, \therefore 四边形 $CONM$ 为矩形.

$\therefore \angle CMN = 90^\circ$, $CO = MN$.

$\because y = x^2 - 4x + 3$, \therefore 点 C 的坐标为 $(0, 3)$ (1分).

$\because B(3, 0)$, $\therefore OB = OC$. $\because \angle COB = 90^\circ$, $\therefore \angle OCB = \angle BCM = 45^\circ$, (1分).

又 $\because \angle ACB = \angle PCB$, $\therefore \angle OCB - \angle ACB = \angle BCM - \angle PCB$, 即 $\angle OCA = \angle PCM$ (1分).

$$\therefore \tan \angle OCA = \tan \angle PCM. \therefore \frac{1}{3} = \frac{PM}{MC}.$$

设 $PM = a$, 则 $MC = 3a$, $PN = 3 - a$.

$$\therefore P(3a, 3-a). \dots\dots\dots (1分)$$

将 $P(3a, 3-a)$ 代入 $y = x^2 - 4x + 3$, 得

$$(3a)^2 - 12a + 3 = 3 - a.$$

$$\text{解得 } a_1 = \frac{11}{9}, a_2 = 0 \text{ (舍)}. \therefore P\left(\frac{11}{3}, \frac{16}{9}\right). \dots\dots\dots (1分)$$

(3) 设抛物线平移的距离为 m . 得 $y = (x-2)^2 - 1 - m$,

∴D 的坐标为 (2, -1-m). (1 分)

过点 D 作直线 EF//x 轴, 交 y 轴于点 E, 交 PQ 的延长线于点 F.

∵∠OED=∠QFD=∠ODQ=90°,

∴∠EOD+∠ODE=90°, ∠ODE+∠QDF=90°,

∴∠EOD=∠QDF, (1 分)

$$\therefore \tan \angle EOD = \tan \angle QDF. \therefore \frac{DE}{OE} = \frac{QF}{DF}. \therefore \frac{2}{m+1} = \frac{\frac{16}{9}-m+1+m}{\frac{11}{3}-2}.$$

解得 $m = \frac{1}{5}$. 所以, 抛物线平移的距离为 $\frac{1}{5}$ (1 分)

25. 解: (1) ∵AD//BC, ∴∠EDQ=∠DBC. (1 分)

$$\therefore \frac{DE}{DQ} = 1, \frac{BD}{BC} = 1, \therefore \frac{DE}{DQ} = \frac{BD}{BC}. \dots\dots\dots (1 \text{ 分})$$

∴△DEQ ∽ △BCD. (1 分)

∴∠DQE=∠BDC, ∴EQ//CD. (1 分)

(2) 设 BP 的长为 x, 则 DQ=x, QP=2x-10. (1 分)

$$\therefore \triangle DEQ \sim \triangle BCD, \therefore \frac{EQ}{DC} = \frac{QD}{CB}, \therefore EQ = \frac{2}{5}x. \dots\dots\dots (1 \text{ 分})$$

(i) 当 EQ=EP 时,

∴∠EQP=∠EPQ,

∵DE=DQ, ∴∠EQP=∠QED, ∴∠EPQ=∠QED,

$$\therefore \triangle EQP \sim \triangle DEQ, \therefore \frac{EQ}{DE} = \frac{QP}{EQ}, \therefore \left(\frac{2}{5}x\right)^2 = (2x-10) \cdot x,$$

解得 $x = \frac{125}{23}$, 或 $x = 0$ (舍去). (2 分)

(ii) 当 QE=QP 时,

$$\therefore \frac{2}{5}x = 2x-10, \text{ 解得 } x = \frac{25}{4}, \dots\dots\dots (1 \text{ 分})$$

∵ $\frac{25}{4} > 6$, ∴此种情况不存在. (1 分)

$$\therefore BP = \frac{125}{23}$$

(3) 过点 P 作 PH⊥EQ, 交 EQ 的延长线于点 H; 过点 B 作 BG⊥DC, 垂足为点 G.

$$\therefore BD=BC, BG \perp DC, \therefore DG=2, BG=4\sqrt{6},$$

$$\therefore BP=DQ=m, \therefore PQ=10-2m.$$

$$\therefore EQ \parallel DC \therefore \angle PQH = \angle BDG. \\ \text{又} \therefore \angle PHQ = \angle BGD = 90^\circ,$$

$$\therefore \triangle PHQ \sim \triangle BGD. \dots\dots\dots (1 \text{ 分})$$

$$\therefore \frac{PH}{BG} = \frac{PQ}{BD} = \frac{HQ}{GD}, \therefore \frac{PH}{4\sqrt{6}} = \frac{10-2m}{10} = \frac{HQ}{2}.$$

$$\therefore HQ = \frac{10-2m}{5}, PH = \frac{2\sqrt{6}(10-2m)}{5}. \dots\dots\dots (2 \text{ 分})$$

$$\therefore EH = \frac{10-2m}{5} + \frac{2m}{5} = 2,$$

$$\therefore \tan \angle PEQ = \frac{PH}{EH} = \frac{2\sqrt{6}(10-2m)}{5} \times \frac{1}{2} = 2\sqrt{6} - \frac{2\sqrt{6}}{5}m. \dots\dots\dots (1 \text{ 分})$$