

九年级数学复习测试卷参考答案

测试卷(一)

一、选择题

1.A 2.C 3.D 4.A 5.D 6.B 7.A 8.B 9.C
10.B

二、填空题

11. $(m-2)^2$ 12. 3 13. $0.8x$ 14. -3 15. $x \geq -1$
16. 3 17. k^{n+2021}

三、解答题

$$\begin{aligned} 18. \text{解: 原式} &= 3 - 2 \times \frac{1}{2} + 8 + 1 \\ &= 3 - 1 + 8 + 1 \\ &= 11. \end{aligned}$$

$$\begin{aligned} 19. \text{解: 原式} &= 4x^2 - 12xy + 9y^2 - 4x^2 + y^2 \\ &= -12xy + 10y^2, \end{aligned}$$

$$\text{当 } x = -\frac{1}{3}, y = \frac{1}{2} \text{ 时,}$$

$$\begin{aligned} \text{原式} &= -12 \times \left(-\frac{1}{3}\right) \times \frac{1}{2} + 10 \times \left(\frac{1}{2}\right)^2 \\ &= 2 + \frac{5}{2} \\ &= 4 \frac{1}{2}. \end{aligned}$$

$$\begin{aligned} 20. \text{解: 原式} &= \left(\frac{x+1}{x-2} \cdot \frac{x-2}{x-2}\right) \div \frac{x(x-2)}{(x-2)^2} \\ &= \frac{3}{x-2} \cdot \frac{x-2}{x} \\ &= \frac{3}{x}, \end{aligned}$$

$$\text{当 } x = \sqrt{3} \text{ 时, 原式} = \frac{3}{\sqrt{3}} = \sqrt{3}.$$

$$21. \text{解: (1) } A + 4B$$

$$\begin{aligned} (2) A + B &= 4x^2 - 4xy + y^2 + x^2 + xy - 5y^2, \\ &= 5x^2 - 3xy - 4y^2, \\ &= 4x^2 + x^2 - 3xy - 4y^2, \\ &= 1 - 3, \\ &= -2. \end{aligned}$$

$$22. \text{解: (1) } P = \frac{2a}{a^2 - b^2} - \frac{1}{a+b} = \frac{2a}{(a+b)(a-b)} - \frac{1}{a+b} =$$

$$\frac{2a - a + b}{(a+b)(a-b)} = \frac{1}{a-b};$$

$$(2) \because \text{点}(a, b) \text{在一次函数 } y = x - \sqrt{2} \text{的图像上,}$$

$$\therefore b = a - \sqrt{2},$$

$$\therefore a - b = \sqrt{2},$$

$$\therefore P = \frac{\sqrt{2}}{2};$$

$$23. \text{解: (1) 原式} = \frac{(\sqrt{5} + \sqrt{3})(\sqrt{5} + \sqrt{3})}{(\sqrt{5} + \sqrt{3})(\sqrt{5} - \sqrt{3})}$$

$$= \frac{8 + 2\sqrt{15}}{2} = 4 + \sqrt{15},$$

$$\text{故答案为: } 4 + \sqrt{15};$$

$$(2) \text{① } \sqrt{7-2\sqrt{10}}$$

$$= \sqrt{(\sqrt{5})^2 + (\sqrt{2})^2 - 2\sqrt{10}}$$

$$= \sqrt{(\sqrt{5} - \sqrt{2})^2} = \sqrt{5} - \sqrt{2};$$

$$\begin{aligned} \text{② 原式} &= \sqrt{2} - 1 + \sqrt{3} - \sqrt{2} + 2 - \sqrt{3} + \dots + \sqrt{2019} - \\ &\sqrt{2018} = \sqrt{2019} - 1. \end{aligned}$$

测试卷(二)

一、选择题

1.A 2.D 3.C 4.B 5.B 6.C 7.D 8.D 9.C
10.B

二、填空题

11. -1 12. 1 13. -2 14. 3 15. 2 16. 250 17.
 $\frac{1}{3}$

三、解答题

$$18. \text{解: } \begin{cases} 2(x+1) > x & \text{①} \\ 1-2x \geq \frac{x+7}{2} & \text{②} \end{cases},$$

$$\text{解①得, } x > -2,$$

$$\text{解②得, } x \leq -1,$$

$$\therefore \text{不等式组的解集为: } -2 < x \leq -1.$$

在数轴上表示为:



$$19. \text{解: } \frac{2}{x+2} + 1 = \frac{x}{x-1},$$

方程两边同时乘以 $(x+2)(x-1)$, 得

$$2(x-1) + (x+2)(x-1) = x(x+2),$$

$$\therefore x = 4,$$

经检验 $x = 4$ 是方程的解;

\therefore 方程的解为 $x = 4$;

$$20. \text{解: } \because a = 1, b = -3, c = -2;$$

$$\therefore b^2 - 4ac = (-3)^2 - 4 \times 1 \times (-2) = 9 + 8 = 17;$$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{3 \pm \sqrt{17}}{2},$$

$$\therefore x_1 = \frac{3 + \sqrt{17}}{2}, x_2 = \frac{3 - \sqrt{17}}{2}.$$

21. 解: 设计划每天生产 x 顶帐篷, 则实际每天生产帐篷 $(1+25\%)x$ 顶,

$$\text{依题意得: } \frac{10000}{x} - 10 = \frac{10000}{(1+25\%)x}.$$

解得 $x = 200$.

经检验 $x = 200$ 是所列方程的解, 且符合题意.

答: 计划每天生产 200 顶帐篷.

22. 解: (1) 设甲物资采购了 x 吨, 乙物资采购了 y 吨,

$$\text{依题意, 得: } \begin{cases} x+y=540 \\ 3x+2y=1380 \end{cases},$$

$$\text{解得: } \begin{cases} x=300 \\ y=240 \end{cases}.$$

答: 甲物资采购了 300 吨, 乙物资采购了 240 吨.

(2) 设安排 A 型卡车 m 辆, 则安排 B 型卡车 $(50-m)$ 辆,

$$\text{依题意, 得: } \begin{cases} 7m+5(50-m) \geq 300 \\ 3m+7(50-m) \geq 240 \end{cases},$$

$$\text{解得: } 25 \leq m \leq 27 \frac{1}{2}.$$

$\therefore m$ 为正整数,

$\therefore m$ 可以为 25, 26, 27,

\therefore 共有 3 种运输方案, 方案 1: 安排 25 辆 A 型卡车, 25 辆 B 型卡车; 方案 2: 安排 26 辆 A 型卡车, 24 辆 B 型卡车; 方案 3: 安排 27 辆 A 型卡车, 23 辆 B 型卡车.

23. 解: (1) 10; 15.

$$(2) y = \frac{x(x-1)}{2}; \quad 1128.$$

$$(3) \text{依题意, 得: } \frac{x(x-1)}{2} = 190,$$

$$\text{化简, 得: } x^2 - x - 380 = 0,$$

$$\text{解得: } x_1 = 20, x_2 = -19 (\text{不合题意, 舍去}).$$

答: 该班共有 20 名女生.

测试卷(三)

一、选择题

1.B 2.D 3.D 4.A 5.C 6.B 7.C 8.A 9.D

10.C

二、填空题

11. -2 12. $x \neq 2$ 13. -9 14. -4 15. 41 16.

(2019, -2) 17. $-3 \leq b \leq 1$

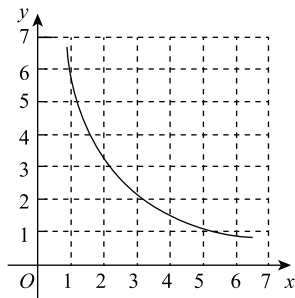
三、解答题

18. 解: (1) 函数图像如图所示, 设函数表达式为

$$y = \frac{k}{x} (k \neq 0),$$

把 $x=1, y=6$ 代入, 得 $k=6$,

$$\therefore \text{函数表达式为 } y = \frac{6}{x} (x > 0);$$



(2) $\because k=6 > 0$,

\therefore 在第一象限, y 随 x 的增大而减小,

$\therefore 0 < x_1 < x_2$ 时, 则 $y_1 > y_2$.

19. 解: (1) 设大货车、小货车各有 x 与 y 辆,

$$\text{由题意可知: } \begin{cases} 15x+10y=260 \\ x+y=20 \end{cases},$$

$$\text{解得: } \begin{cases} x=12 \\ y=8 \end{cases},$$

答: 大货车、小货车各有 12 辆与 8 辆.

(2) 设到 A 地的大货车有 x 辆, 则到 A 地的小货车有 $(10-x)$ 辆,

到 B 地的大货车有 $(12-x)$ 辆, 到 B 地的小货车有 $(x-2)$ 辆,

$$\therefore y = 900x + 500(10-x) + 1000(12-x) + 700(x-2) = 100x + 15600,$$

其中 $2 < x < 10$.

(3) 运往 A 地的物资共有 $[15x + 10(10-x)]$ 吨,

$$15x + 10(10-x) \geq 140,$$

$$\text{解得: } x \geq 8,$$

$$\therefore 8 \leq x < 10,$$

当 $x=8$ 时,

y 有最小值, 此时 $y = 100 \times 8 + 15600 = 16400$ 元,

答: 总运费最小值为 16400 元.

20. 解: (1) \because 点 A 在反比例函数 $y = \frac{4}{x}$ 上,

$$\therefore \frac{4}{x} = 4, \text{ 解得 } m=1,$$

\therefore 点 A 的坐标为 $(1, 4)$,

又 \because 点 B 也在反比例函数 $y = \frac{4}{x}$ 上,

$$\therefore \frac{4}{2} = n, \text{ 解得 } n = 2,$$

\therefore 点 B 的坐标为 $(2, 2)$,

又 \because 点 A, B 在 $y = kx + b$ 的图像上,

$$\therefore \begin{cases} k + b = 4 \\ 2k + b = 2 \end{cases}, \text{ 解得 } \begin{cases} k = -2 \\ b = 6 \end{cases},$$

\therefore 一次函数的解析式为 $y = -2x + 6$.

(2) 根据图像得: $kx + b - \frac{4}{x} > 0$ 时, x 的取值范围

为 $x < 0$ 或 $1 < x < 2$;

(3) \because 直线 $y = -2x + 6$ 与 x 轴的交点为 N ,

\therefore 点 N 的坐标为 $(3, 0)$,

$$S_{\triangle AOB} = S_{\triangle AON} - S_{\triangle BON} = \frac{1}{2} \times 3 \times 4 - \frac{1}{2} \times 3 \times 2 = 3.$$

21. 解: (1) 在矩形 $ABCD$ 中, $AB = 3, AD = 8$,

$\therefore CD = AB = 3, BC = AD = 8$,

$\therefore D(-6, 0)$,

$\therefore A(-6, 8), C(-3, 0), B(-3, 8)$,

$\because E$ 是 BC 的中点,

$\therefore E(-3, 4)$,

\because 点 E 在反比例函数 $y = \frac{k}{x}$ 的图像上,

$\therefore k = -3 \times 4 = -12$,

设经过 A, E 两点的一次函数的表达式为 $y = k'x + b$,

$$\therefore \begin{cases} -6k' + b = 8 \\ -3k' + b = 4 \end{cases},$$

$$\therefore \begin{cases} k' = -\frac{4}{3} \\ b = 0 \end{cases},$$

\therefore 经过 A, E 两点的一次函数的表达式为 $y = -\frac{4}{3}x$;

(2) 如图 1, 由 (1) 知, $k = -12$,

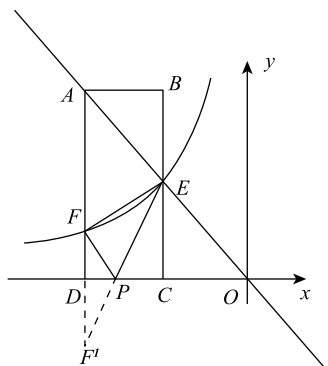


图1

\therefore 反比例函数的解析式为 $y = -\frac{12}{x}$,

\therefore 点 F 的横坐标为 -6 ,

\therefore 点 F 的纵坐标为 2 ,

$\therefore F(-6, 2)$,

作点 F 关于 x 轴的对称点 F' , 则 $F'(-6, -2)$, 连接 EF' 交 x 轴于 P , 此时, $PE + PF$ 的值最小,

$\therefore E(-3, 4)$,

\therefore 直线 EF' 的解析式为 $y = 2x + 10$,

令 $y = 0$, 则 $2x + 10 = 0$,

$\therefore x = -5$,

$\therefore P(-5, 0)$;

(3) 如图 2,

由 (2) 知, $F'(-6, -2)$,

$\therefore E(-3, 4), F(-6, 2)$,

$$\therefore S_{\triangle PEF} = S_{\triangle EFF'} - S_{\triangle PFF'} = \frac{1}{2} \times (2+2) \times (-3+6) - \frac{1}{2}$$

$$(2+2) \times (-5+6) = 4,$$

$\therefore E(-3, 4), F(-6, 2)$,

\therefore 直线 EF 的解析式为 $y = \frac{2}{3}x + 6$,

由 (1) 知, 经过 A, E 两点的一次函数的表达式为 $y = -\frac{4}{3}x$,

设点 $Q(m, -\frac{4}{3}m)$,

过点 Q 作 y 轴的平行线交 EF 于 G ,

$\therefore G(m, \frac{2}{3}m + 6)$,

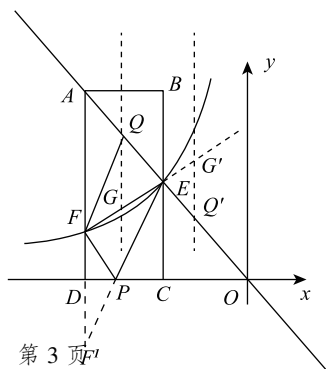
$$\therefore QG = |-\frac{4}{3}m - \frac{2}{3}m - 6| = |2m + 6|,$$

$$\therefore S_{\triangle QEF} = S_{\triangle PEF},$$

$$\therefore S_{\triangle QEF} = \frac{1}{2} |2m + 6| \times (-3 + 6) = 4,$$

$$\therefore m = -\frac{5}{3} \text{ 或 } m = -\frac{13}{3},$$

$$\therefore Q\left(-\frac{5}{3}, \frac{20}{9}\right) \text{ 或 } \left(-\frac{13}{3}, \frac{52}{9}\right).$$



测试卷(四)

一、选择题

- 1.C 2.B 3.B 4.C 5.B 6.A 7.C 8.D 9.B
10.D

二、填空题

11. $m \neq 2$ 12. $y = x^2 + 6x$ 13. 直线 $x = \frac{1}{2}$ 14. $y_3 > y_2 > y_1$ 15. $y = 3(x+4)^2$ 16. $y = 2(x-2)^2 + 1$ (答案不唯一) 17. 8

三、解答题

18. 解: (1) \because 垂直于墙的边长为 x ,
 \therefore 平行于墙的边长为 $40-2x$,
 $\therefore y = x(40-2x)$,
即 y 与 x 之间的函数关系式为 $y = -2x^2 + 40x$;

$$(2) \text{由题意, 得} \begin{cases} x > 0 \\ 40-2x > 0 \end{cases}$$

解得 $0 < x < 20$.

19. 解: \because 抛物线 $y = ax^2 + bx + 6$ 经过点 $A(-2, 0)$, $B(4, 0)$,

$$\therefore \begin{cases} 4a - 2b + 6 = 0 \\ 16a + 4b + 6 = 0 \end{cases}$$

$$\text{解得} \begin{cases} a = -\frac{3}{4} \\ b = \frac{3}{2} \end{cases},$$

$$\therefore \text{抛物线的函数表达式为 } y = -\frac{3}{4}x^2 + \frac{3}{2}x + 6$$

20. 解: (1) $y = x^2 - 6x + 9 - 9 + 5 = (x-3)^2 - 4$, 即 $y = (x-3)^2 - 4$;

(2) 由(1)知, 抛物线解析式为 $y = (x-3)^2 - 4$,
所以抛物线的对称轴为: $x = 3$, 顶点坐标为 $(3, -4)$.

21. 解: (1) 设 $y = kx + b$,

由表可知: 当 $x = 15$ 时, $y = 150$, 当 $x = 16$ 时, $y = 140$,

$$\text{则} \begin{cases} 150 = 15k + b \\ 140 = 16k + b \end{cases}, \text{解得: } \begin{cases} k = -10 \\ b = 300 \end{cases},$$

$\therefore y$ 关于 x 的函数解析式为: $y = -10x + 300$;

(2) 由题意可得:

$$w = (-10x + 300)(x - 11) = -10x^2 + 410x - 3300,$$

$\therefore w$ 关于 x 的函数解析式为: $w = -10x^2 + 410x - 3300$;

$$(3) \because \text{对称轴 } x = \frac{410}{-2 \times (-10)} = 20.5, a = -10 < 0, x$$

是整数,

$\therefore x = 20$ 或 21 时, w 有最大值,

当 $x = 20$ 或 21 时, 代入, 可得: $w = 900$,

\therefore 该工艺品每件售价为 20 元或 21 元时, 工艺品厂试销该工艺品每天获得的利润最大, 最大利润是 900 元.

22. 解: (1) \because 抛物线 $y = ax^2 + bx + c$ 的顶点是 $A(1, 3)$,

\therefore 抛物线的解析式为 $y = a(x-1)^2 + 3$,

$\because OA$ 绕点 O 顺时针旋转 90° 后得到 OB ,

$\therefore B(3, -1)$,

把 $B(3, -1)$ 代入 $y = a(x-1)^2 + 3$ 可得 $a = -1$,

\therefore 抛物线的解析式为 $y = -(x-1)^2 + 3$, 即 $y = -x^2 + 2x + 2$,

(2) ①如图 1 中, 连接 OA' , $A'B$.

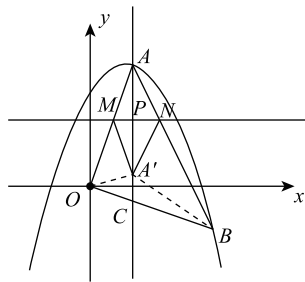


图 1

$\therefore B(3, -1)$,

\therefore 直线 OB 的解析式为 $y = -\frac{1}{3}x$,

$\therefore A(1, 3)$,

$\therefore C(1, -\frac{1}{3})$,

$\therefore P(1, m)$, $AP = PA'$,

$\therefore A'(1, 2m-3)$,

由题意 $3 > 2m-3 > -\frac{1}{3}$,

$\therefore 3 > m > \frac{4}{3}$.

②当点 P 在 x 轴上方时, \because 直线 OA 的解析式为 $y = 3x$, 直线 AB 的解析式为 $y = -2x + 5$,

$\therefore P(1, m)$,

$\therefore M(\frac{m}{3}, m)$, $N(\frac{5-m}{2}, m)$,

$$\therefore MN = \frac{5-m}{2} - \frac{m}{3} = \frac{15-5m}{6},$$

$$\therefore S_{\triangle A'MN} = \frac{5}{6} S_{\triangle OA'B},$$

$$\therefore \frac{1}{2} \cdot (m-2m+3) \cdot \frac{15-5m}{6} = \frac{5}{6} \times \frac{1}{2} \times |2m-3| +$$

$$\frac{1}{3} \times 3,$$

$$\text{整理得 } m^2 - 6m + 9 = |6m - 8|$$

$$\text{解得 } m = 6 + \sqrt{19} \text{ (舍去) 或 } 6 - \sqrt{19},$$

$$\text{当点 } P \text{ 在 } x \text{ 轴下方时, 同法可得 } \frac{1}{2} \cdot (3-m) \cdot$$

$$\left(\frac{5-m}{2} + 3m\right) = \frac{5}{6} \times \frac{1}{2} \times \left[-\frac{1}{3} - (2m-3)\right] \times 3,$$

$$\text{整理得: } 3m^2 - 12m - 1 = 0,$$

$$\text{解得 } m = \frac{6 - \sqrt{39}}{3} \text{ 或 } \frac{6 + \sqrt{39}}{3} \text{ (舍去),}$$

$$\therefore \text{ 满足条件的 } m \text{ 的值为 } 6 - \sqrt{19} \text{ 或 } \frac{6 - \sqrt{39}}{3}.$$

测试卷(五)

一、选择题

1.B 2.D 3.A 4.D 5.C 6.B 7.A 8.C 9.B
10.A

二、填空题

11. $\frac{1}{2}$ 12. $\frac{2}{5}$ 13. 90 14. 80 15. 5 16. 4 17. 2

三、解答题

18. 解: (1) 11; 75; 78.

(2) 处于 B 等级及以上的人数为: $\frac{7+1}{20} \times 3000 = 1200$;

(3) 该区教师每人一年(按 365 天计算)平均阅读文章的篇数 = $\frac{1}{3} \times \frac{78.5}{5} \times 365 \approx 1910$.

19. 解: (1) \because 这次抽样中, “空气质量不低于良”的频数是 $30 - 0 - 1 - 2 = 27$,

$$\therefore \text{ 频率为 } \frac{27}{30} = 0.9;$$

$$(2) \because a = 30 - (15 + 2 + 1) = 12,$$

$$\therefore 365 \times \frac{12}{30} = 146.$$

答: 2009 年全年(共 365 天)空气质量为优的天数大约为 146 天.

20. 解: (1) 这四名候选人面试成绩的中位数为:

$$\frac{88+90}{2} = 89 \text{ (分)};$$

$$(2) \text{ 由题意得, } x \times 60\% + 90 \times 40\% = 87.6$$

$$\text{解得 } x = 86,$$

答: 表中 x 的值为 86;

$$(3) \text{ 甲候选人的综合成绩为: } 90 \times 60\% + 88 \times 40\% = 89.2 \text{ (分)},$$

$$\text{乙候选人的综合成绩为: } 84 \times 60\% + 92 \times 40\% =$$

$$87.2 \text{ (分)},$$

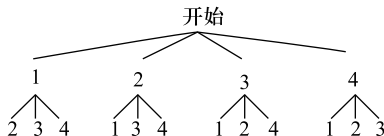
$$\text{丁候选人的综合成绩为: } 88 \times 60\% + 86 \times 40\% =$$

$$87.2 \text{ (分)},$$

\therefore 以综合成绩排序确定所要招聘的前两名的
人选是甲和丙.

21. 解: (1) 小刚摸出的小球上的数字是 4 的概率
是 $\frac{1}{3}$;

(2) 解: (1) 画树状图得:



\therefore 共有 12 种可能的结果数, 即点 P 所有可能的
坐标为 $(1, 2), (1, 3), (1, 4), (2, 1), (2, 3),$
 $(2, 4), (3, 1), (3, 2), (3, 4), (4, 1), (4, 2),$
 $(4, 3)$;

点 $P(x, y)$ 在函数 $y = -x + 6$ 的图像上的结果有
2 个,

\therefore 点 $P(x, y)$ 在函数 $y = -x + 6$ 的图像上的概率
为 $\frac{2}{12} = \frac{1}{6}$.

22. 解: (1) 喜欢用电话沟通的人数为 20, 所占百
分比为 20%,

\therefore 此次共抽查了: $20 \div 20\% = 100$ 人,

喜欢用 QQ 沟通所占比例为: $\frac{30}{100} = \frac{3}{10}$,

\therefore “QQ” 的扇形圆心角的度数为: $360^\circ \times \frac{3}{10} =$
108,

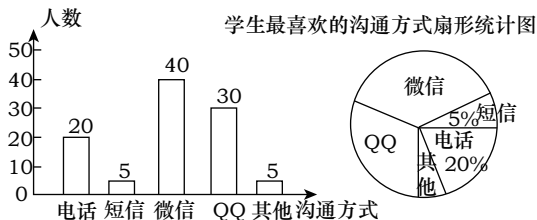
故答案为: 100、108°;

(2) 喜欢用短信的人数为: $100 \times 5\% = 5$ 人

喜欢用微信的人数为: $100 - 20 - 5 - 30 - 5 = 40$ 人

补充图形, 如图所示:

学生最喜欢的沟通方式条形统计图

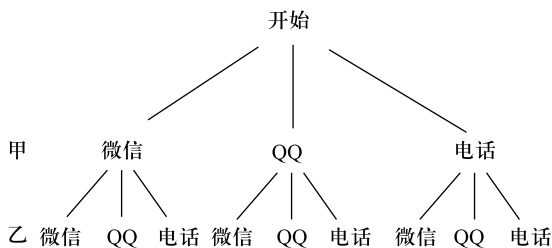


(3) 喜欢用微信沟通所占百分比为: $\frac{40}{100} \times 100\%$
= 40%

\therefore 该校共有 2500 名学生, 请估计该校最喜欢
用“微信”进行沟通的学生有: $2500 \times 40\% =$

1000 人;

(4) 画出树状图, 如图所示



所有情况共有 9 种情况, 其中甲、乙两名同学恰好选择同一种沟通方式的共有 3 种情况, 故甲、乙两名同学恰好选中同一种沟通方式的

$$\text{概率} = \frac{3}{9} = \frac{1}{3}.$$

测试卷(六)

一、选择题

1.D 2.B 3.A 4.B 5.C 6.D 7.A 8.B 9.C

10.B

二、填空题

11. 36° 12. 34° 13. 4 14. 9 cm 15. 20 16. $2\sqrt{5}$

17. 1010

三、解答题

18. 证明: $\because BE=CF$,

$$\therefore BE+EF=CF+EF, \text{ 即 } BF=CE,$$

在 $\triangle ABF$ 和 $\triangle DCE$ 中,

$$\begin{cases} AB=DC \\ \angle B=\angle C, \\ BF=CE \end{cases}$$

$$\therefore \triangle ABF \cong \triangle DCE (SAS)$$

$$\therefore AF=DE.$$

19. 解: (1) 由题意可知, $AB = \sqrt{2^2+6^2} = 2\sqrt{10}$, AC

$$= \sqrt{2^2+6^2} = 2\sqrt{10},$$

$$BC = \sqrt{4^2+8^2} = 4\sqrt{5}$$

(2) 连接 AD

$$\text{由 (1) 可知, } AB^2+AC^2=BC^2, AB=AC$$

$$\therefore \angle BAC=90^\circ, \text{ 且 } \triangle ABC \text{ 是等腰直角三角形}$$

$$\therefore \text{以点 } A \text{ 为圆心的 } \widehat{EF} \text{ 与 } BC \text{ 相切于点 } D$$

$$\therefore AD \perp BC$$

$$\therefore AD = \frac{1}{2}BC = 2\sqrt{5} \text{ (或用等面积法 } AB \cdot AC =$$

$$BC \cdot AD \text{ 求出 } AD \text{ 长度)}$$

$$\therefore S_{\text{阴影}} = S_{\triangle ABC} - S_{\text{扇形 } EAF}$$

$$S_{\triangle ABC} = \frac{1}{2} \times 2\sqrt{10} \times 2\sqrt{10} = 20$$

$$S_{\text{扇形 } EAF} = \frac{1}{4}\pi (2\sqrt{5})^2 = 5\pi$$

$$\therefore S_{\text{阴影}} = 20 - 5\pi$$

20. 解: (1) 四边形 $ABB'A'$ 是菱形.

理由: 由平移得 $AA' \parallel BB'$, $AA' = BB'$,

\therefore 四边形 $ABB'A'$ 是平行四边形, $\angle AA'B = \angle A'BC$.

$$\therefore BA' \text{ 平分 } \angle ABC,$$

$$\therefore \angle ABA' = \angle A'BC,$$

$$\therefore \angle AA'B = \angle A'BA.$$

$$\therefore AB = AA',$$

\therefore 四边形 $ABB'A'$ 是菱形;

(2) 解: 过点 A 作 $AF \perp BC$ 于点 F

由 (1) 得 $BB' = BA = 6$.

$$\therefore AC' \perp A'B',$$

$$\therefore \angle B'EC' = 90^\circ,$$

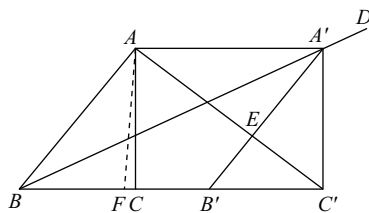
$$\therefore AB \parallel A'B',$$

$$\therefore \angle BAC' = \angle B'EC' = 90^\circ.$$

$$\text{在 Rt } \triangle ABC' \text{ 中, } AC' = \sqrt{BC'^2 - AB^2} = 8.$$

$$\therefore S_{\triangle ABC'} = \frac{1}{2}AB \cdot AC' = \frac{1}{2}BC' \cdot AF,$$

$$\therefore AF = \frac{AB \cdot AC'}{BC'} = \frac{24}{5},$$



$$\therefore S_{\text{菱形 } ABB'A'} = BB' \cdot AF = \frac{144}{5},$$

$$\therefore \text{菱形 } ABB'A' \text{ 的面积是 } \frac{144}{5}.$$

21. (1) 证明: $\because AE \perp BP$,

$$\therefore \angle DBE + \angle DEB = 90^\circ,$$

$$\therefore \angle ACB = 90^\circ,$$

$$\therefore \angle DBE + \angle CPB = 90^\circ,$$

$$\therefore \angle CPB = \angle DEB,$$

在 $\triangle ACE$ 和 $\triangle BCP$ 中,

$$\begin{cases} \angle ACE = \angle BCP \\ \angle AEC = \angle BPC, \\ AC = BC \end{cases}$$

$$\therefore \triangle ACE \cong \triangle BCP (AAS);$$

(2) 解: 在 $\text{Rt } \triangle ABC$ 中, $AB = \sqrt{AC^2 + BC^2} = 2$,

$$\therefore AD = CD,$$

$\therefore \angle DAC = \angle DCA$,
 $\therefore \angle DAC + \angle DEC = 90^\circ$, $\angle DCE + \angle DCA = 90^\circ$,
 $\therefore \angle DCE = \angle DEC$,
 $\therefore DC = DE$,
 $\therefore AD = DE$,
 $\therefore AD = DE$, $BD \perp AE$,
 $\therefore BE = AB = 2$,
 $\therefore \triangle ACE \cong \triangle BCP$,

$$\therefore CP = CE = BE - BC = 2 - \sqrt{2};$$

(3) 解: $\angle ADC$ 的大小保持不变,
 理由如下: 作 $CF \perp BD$ 于 F , $CH \perp AE$ 于 H ,

$\therefore \triangle ACE \cong \triangle BCP$,
 $\therefore CE = CP$, $\angle BPC = \angle E$,
 在 $\triangle CFP$ 和 $\triangle CHE$ 中,

$$\begin{cases} \angle CFP = \angle CHE \\ \angle FPC = \angle HEC, \\ CP = CE \end{cases}$$

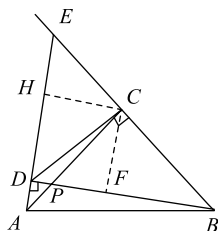
$$\therefore \triangle CFP \cong \triangle CHE (AAS)$$

$\therefore CF = CH$, 又 $CF \perp BD$, $CH \perp AE$,

$\therefore CD$ 平分 $\angle EDB$,

$$\therefore \angle EDC = \frac{1}{2} \angle EDB = 45^\circ,$$

$\therefore \angle ADC = 180^\circ - \angle EDC = 135^\circ$, 即 $\angle ADC$ 的大小保持不变, 为 135° .



22. 应用: 解: ①若 $PB = PC$, 连接 PB , 则 $\angle PCB = \angle PBC$,

$\therefore CD$ 为等边三角形的高,

$\therefore AD = BD$, $\angle PCB = 30^\circ$,

$\therefore \angle PBD = \angle PBC = 30^\circ$,

$$\therefore PD = \frac{\sqrt{3}}{3} DB = \frac{\sqrt{3}}{6} AB,$$

与已知 $PD = \frac{1}{2} AB$ 矛盾, $\therefore PB \neq PC$,

②若 $PA = PC$, 连接 PA , 同理可得 $PA \neq PC$,

③若 $PA = PB$, 由 $PD = \frac{1}{2} AB$, 得 $PD = BD$,

$$\therefore \angle APD = 45^\circ,$$

故 $\angle APB = 90^\circ$;

探究: 解: $\because BC = 5, AB = 3$,

$$\therefore AC = \sqrt{BC^2 - AB^2} = \sqrt{5^2 - 3^2} = 4,$$

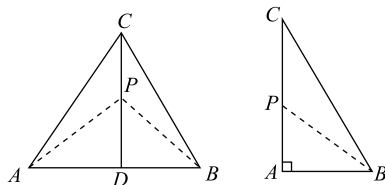
①若 $PB = PC$, 设 $PA = x$, 则 $x^2 + 3^2 = (4 - x)^2$,

$$\therefore x = \frac{7}{8}, \text{ 即 } PA = \frac{7}{8},$$

②若 $PA = PC$, 则 $PA = 2$,

③若 $PA = PB$, 由图知, 在 $\text{Rt} \triangle PAB$ 中, 不可能.

故 $PA = 2$ 或 $\frac{7}{8}$.



测试卷(七)

一、选择题

1. B 2. A 3. C 4. C 5. A 6. D 7. D 8. D 9. D

10. A

二、填空题

11. 300 12. 16:81 13. $\frac{20}{3}$ 14. $\sqrt{5} - 1$ 15. -2

16. (2, 4) 或 (-2, -4) 17. $20\sqrt{2}$

三、解答题

18. 解: (1) 证明: $\because \angle A = \angle A$, $\angle ACD = \angle B$,

$$\therefore \triangle ABC \sim \triangle ACD.$$

(2) 解: $\because \triangle ABC \sim \triangle ACD$,

$$\therefore \frac{AC}{AD} = \frac{AB}{AC},$$

$$\therefore \frac{6}{4} = \frac{AB}{6},$$

$$\therefore AB = 9,$$

$$\therefore BD = AB - AD = 9 - 4 = 5.$$

19. 解: (1) $\triangle ACF \sim \triangle AHC$. 理由如下:

$$\because AC = \sqrt{2}, AF = 1, AH = 2,$$

$$\therefore \frac{AH}{AC} = \frac{AC}{AF} = \sqrt{2},$$

而 $\angle FAC = \angle CAH$,

$$\therefore \triangle ACF \sim \triangle AHC;$$

(2) $\because \triangle ACF \sim \triangle AHC$

$$\therefore \angle 2 = \angle ACH,$$

而 $\angle 1 = \angle ACH + \angle 3$,

$$\therefore \angle 1 = \angle 2 + \angle 3.$$

$$\therefore \angle 1 = 45^\circ,$$

$$\therefore \angle 1 + \angle 2 + \angle 3 = 90^\circ.$$

20. \because 四边形 $EGHF$ 为正方形

$$\therefore BC \parallel EF$$

$$\therefore \triangle AEF \sim \triangle ABC$$

设正方形零件边长为 x mm, 则 $KD=EF=x$ mm

$$AK=(80-x)\text{ mm}$$

$$\therefore AD \perp BC$$

$$\therefore \frac{EF}{BC} = \frac{AK}{AD}$$

$$\therefore \frac{x}{120} = \frac{80-30}{80}$$

$$\therefore x=48$$

\therefore 正方形零件边长为 48 mm.

21. 解: 设楼高 CE 为 x 米,

$$\therefore \text{在 Rt} \triangle AEC \text{ 中}, \angle CAE=45^\circ,$$

$$\therefore AE=CE=x,$$

$$\therefore AB=20,$$

$$\therefore BE=x-20,$$

$$\text{在 Rt} \triangle CEB \text{ 中}, CE=BE \cdot \tan 63.4^\circ \approx 2(x-20),$$

$$\therefore 2(x-20)=x,$$

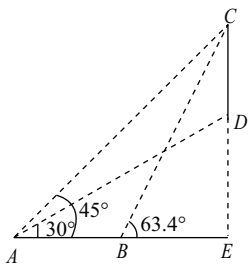
$$\text{解得: } x=40(\text{米}),$$

$$\text{在 Rt} \triangle DAE \text{ 中}, DE=AE \tan 30^\circ = 40 \times \frac{\sqrt{3}}{3} =$$

$$\frac{40\sqrt{3}}{3},$$

$$\therefore CD=CE-DE=40-\frac{40\sqrt{3}}{3} \approx 17(\text{米}),$$

答: 大楼部分楼体 CD 的高度约为 17 米.



图②

22. 解: (1) 如图作 $DH \perp AB$ 于 H , 则四边形 $DHBC$ 是矩形,

$$\therefore CD=BH=8, DH=BC=6,$$

$$\therefore AH=AB-BH=8, AD=\sqrt{DH^2+AH^2}=10, BD=\sqrt{CD^2+BC^2}=10,$$

$$\text{由题意 } AP=AD-DP=10-2t.$$

(2) 当以点 A, P, Q 为顶点的三角形与 $\triangle ABD$ 相似时,

$$\therefore \frac{AP}{AD} = \frac{AQ}{AB} \text{ 或 } \frac{AP}{AB} = \frac{AQ}{AD},$$

$$\therefore \frac{10-2t}{10} = \frac{2t}{16} \text{ 或 } \frac{10-2t}{16} = \frac{2t}{10},$$

$$\text{解得: } t = \frac{40}{13} \text{ 或 } t = \frac{25}{13},$$

$$\therefore \text{当 } t = \frac{40}{13} \text{ 或 } t = \frac{25}{13} \text{ 时, 当以点 } A, P, Q \text{ 为顶点的}$$

三角形与 $\triangle ABD$ 相似;

(3) 过 P 作 $PN \perp AB$ 于 N ,

$$\text{当 } PQ \perp BD \text{ 时}, \angle PQN + \angle DBA = 90^\circ,$$

$$\therefore \angle QPN + \angle PQN = 90^\circ,$$

$$\therefore \angle QPN = \angle DBA,$$

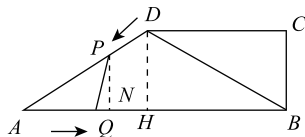
$$\therefore \tan \angle QPN = \frac{QN}{PN} = \frac{3}{4},$$

$$\therefore \frac{\frac{4}{5}(10-2t)-2t}{\frac{3}{5}(10-2t)} = \frac{3}{4},$$

$$\text{解得 } t = \frac{35}{27},$$

$$\text{经检验: } t = \frac{35}{27} \text{ 是分式方程的解,}$$

$$\therefore \text{当 } t = \frac{35}{27} \text{ s 时, } PQ \perp BD.$$



测试卷(八)

一、选择题

1.D 2.A 3.B 4.C 5.C 6.B 7.A 8.B 9.A

10.B

二、填空题

11.7 12.4 13.30° 14.45 15. $\frac{120}{13}$ 16.3 $\sqrt{2}$

17.①②④

三、解答题

18. 证明: (1) $\because \angle BAC = \angle DCA,$

$$\therefore AB \parallel CD,$$

$$\text{又} \because AB=CD,$$

$$\therefore \text{四边形 } ABCD \text{ 为平行四边形;}$$

(2) \because 四边形 $ABCD$ 为平行四边形,

$$\therefore AE=EC=2, BE=DE, AB=CD=5,$$

$$\therefore BC=\sqrt{AB^2-AC^2}=\sqrt{25-16}=3,$$

$$\therefore BE=\sqrt{BC^2+CE^2}=\sqrt{9+4}=\sqrt{13},$$

$$\therefore BD=2BE=2\sqrt{13}.$$

19. (1) 证明: \because 四边形 $ABCD$ 是正方形,

$$\therefore AB=CB, \angle ABC=90^\circ,$$

$\therefore \triangle EBF$ 是等腰直角三角形, 其中 $\angle EBF = 90^\circ$,

$\therefore BE = BF$,

$\therefore \angle ABC - \angle CBF = \angle EBF - \angle CBF$,

$\therefore \angle ABF = \angle CBE$.

在 $\triangle ABF$ 和 $\triangle CBE$ 中, 有
$$\begin{cases} AB = CB \\ \angle ABF = \angle CBE, \\ BF = BE \end{cases}$$

$\therefore \triangle ABF \cong \triangle CBE$ (SAS).

(2) 解: $\triangle CEF$ 是直角三角形. 理由如下:

$\therefore \triangle EBF$ 是等腰直角三角形,

$\therefore \angle BFE = \angle FEB = 45^\circ$,

$\therefore \angle AFB = 180^\circ - \angle BFE = 135^\circ$,

又 $\therefore \triangle ABF \cong \triangle CBE$,

$\therefore \angle CEB = \angle AFB = 135^\circ$,

$\therefore \angle CEF = \angle CEB - \angle FEB = 135^\circ - 45^\circ = 90^\circ$,

$\therefore \triangle CEF$ 是直角三角形.

20. (1) 证明: $\therefore AE \parallel BD, AE = BD$,

\therefore 四边形 $AEBD$ 是平行四边形,

$\therefore AB = AC, D$ 为 BC 的中点,

$\therefore AD \perp BC$,

$\therefore \angle ADB = 90^\circ$,

\therefore 四边形 $AEBD$ 是矩形.

(2) 解: \therefore 四边形 $AEBD$ 是矩形,

$\therefore \angle AEB = 90^\circ$,

$\therefore AE = 2, BE = 2\sqrt{3}$,

$\therefore BC = 4$,

$\therefore EC = \sqrt{BE^2 + BC^2} = 2\sqrt{7}$

$\therefore AE \parallel BC$,

$\therefore \triangle AEF \sim \triangle BCF$,

$\therefore \frac{EF}{CF} = \frac{AE}{BC} = \frac{1}{2}$,

$\therefore EF = \frac{1}{3}EC = \frac{2\sqrt{7}}{3}$.

21. (1) 证明: 由题意可得,

$\triangle BCE \cong \triangle BFE, \therefore \angle BEC = \angle BEF, FE = CE$.

$\therefore FG \parallel CE, \therefore \angle FGE = \angle CEB, \therefore \angle FGE = \angle FEC, \therefore FG = FE$,

$\therefore FG = EC$,

\therefore 四边形 $CEFG$ 是平行四边形.

又 $\therefore CE = FE, \therefore$ 四边形 $CEFG$ 是菱形

(2) \therefore 矩形 $ABCD$ 中, $AB = 6, AD = 10, BC = BF$,

$\therefore \angle BAF = 90^\circ, AD = BC = BF = 10$,

$\therefore AF = 8, \therefore DF = 2$.

设 $EF = x$, 则 $CE = x, DE = 6 - x$,

$\therefore FDE = 90^\circ$,

$\therefore 2^2 + (6 - x)^2 = x^2$,

解得 $x = \frac{10}{3}$

$\therefore CE = \frac{10}{3}$,

\therefore 四边形 $CEFG$ 的面积是: $CE \cdot DF = \frac{10}{3} \times 2 =$

$\frac{20}{3}$.

22. 解: (1) \therefore 四边形 $ABCD$ 是对余四边形,

依题意得, $\angle B + \angle D = 90^\circ$,

$\therefore \angle D = 30^\circ$,

$\therefore \angle B = 90^\circ - \angle D = 60^\circ$,

$\therefore AB = AC$,

$\therefore \triangle ABC$ 是等边三角形,

$\therefore \angle ACB = 60^\circ$,

$\therefore \angle ACD = 105^\circ$,

$\therefore \angle BCD = \angle ACB + \angle ACD = 165^\circ$,

在四边形 $ABCD$ 中, $\angle BAD = 360^\circ - \angle B - \angle ACD - \angle D = 360^\circ - 60^\circ - 165^\circ - 30^\circ = 105^\circ$;

(2) 四边形 $ABCD$ 为对余四边形,

证明: $\therefore AD \perp BD$,

$\therefore \angle ADB = 90^\circ$,

$\therefore DA = DB$,

$\therefore \angle BAD = \angle ABD = 45^\circ$,

如图 2, 过点 D 作 $DM \perp CD$, 使 $CD = CM$, 连接 CM, BM ,

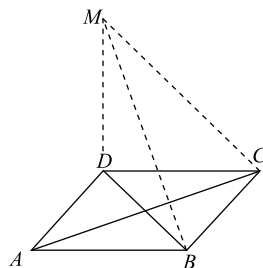


图2

$\therefore \angle DMC = \angle DCM = 45^\circ$,

$\therefore \angle ADB = \angle CDM = 90^\circ$,

$\therefore \angle ADB + \angle BDC = \angle CDM + \angle BDC$,

$\therefore \angle ADC = \angle BDM$.

在 $\triangle ADC$ 和 $\triangle BDM$ 中,

$$\begin{cases} DA = DB \\ \angle ADC = \angle BDM, \\ DC = DM \end{cases}$$

$\therefore \triangle ADC \cong \triangle BDM$ (SAS),

$$\therefore AC=BM.$$

在 $\text{Rt}\triangle MDC$ 中, 根据勾股定理得, $CM^2 = CD^2 + DM^2 = 2CD^2$,

$$\therefore 2CD^2 + CB^2 = AC^2,$$

$$\therefore CM^2 + CB^2 = BM^2,$$

$\therefore \triangle BCM$ 是直角三角形, 且 $\angle BCM = 90^\circ$,

$$\therefore \angle DCM = 45^\circ,$$

$$\therefore \angle DCB = \angle BCM - \angle DCM = 45^\circ,$$

$$\therefore \angle DCB + \angle DAB = 90^\circ,$$

\therefore 四边形 $ABCD$ 为对余四边形;

测试卷(九)

一、选择题

1.B 2.B 3.A 4.C 5.D 6.A 7.C 8.B

9.A 10.B

二、填空题

11. 以点 O 为圆心, 以 8 cm 为半径的圆 12. 90 cm

13. $(3, 3\sqrt{3})$ 14. 6 15. 21π 16. 12 17. $\frac{1+\sqrt{3}}{2}a$

三、解答题

18. (1) 证明: 连接 OC ,

$$\therefore \widehat{AC} = \widehat{BC},$$

$\therefore \angle AOC = \angle BOC$, 又 $CD \perp OA, CE \perp OB$,

$$\therefore CD = CE;$$

(2) 解: $\therefore \angle AOB = 120^\circ$,

$$\therefore \angle AOC = \angle BOC = 60^\circ,$$

$$\therefore \angle CDO = 90^\circ,$$

$$\therefore \angle OCD = 30^\circ,$$

$$\therefore OD = \frac{1}{2}OC = 1,$$

$$\therefore CD = \sqrt{OC^2 - OD^2} = \sqrt{2^2 - 1^2} = \sqrt{3},$$

$$\therefore \triangle OCD \text{ 的面积} = \frac{1}{2} \times OD \times CD = \frac{\sqrt{3}}{2},$$

$$\text{同理可得, } \triangle OCE \text{ 的面积} = \frac{1}{2} \times OE \times CE = \frac{\sqrt{3}}{2},$$

$$\therefore \text{四边形 } DOEC \text{ 的面积} = \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} = \sqrt{3}.$$

19. (1) 解: \therefore 点 C 以 AD 为直径的圆上,

$$\therefore \angle ACD = 90^\circ,$$

$$\text{根据勾股定理得, } AD = \sqrt{AC^2 + CD^2} = \sqrt{3^2 + 1^2} = \sqrt{10},$$

\therefore 点 E 以 AD 为直径的圆上,

$$\therefore \angle AED = 90^\circ,$$

在 $\text{Rt}\triangle ADE$ 中, $\angle DAE = 30^\circ$,

$$\therefore \sin \angle DAE = \frac{DE}{AD},$$

$$\therefore DE = AD \cdot \sin \angle DAE = \sqrt{10} \times \sin 30^\circ = \frac{\sqrt{10}}{2};$$

$$(2) \therefore AF \parallel CD, \therefore \angle ADC = \angle FAD,$$

$$\therefore \angle ADC = \angle AEC, \therefore \angle AEC = \angle FAD,$$

$$\therefore \angle ACE = \angle ADF,$$

$$\therefore \triangle AEC \sim \triangle FAD;$$

20. (1) 证明: $\therefore AB$ 是 $\odot O$ 的直径,

$$\therefore \angle ADB = 90^\circ,$$

$$\therefore OC \parallel BD,$$

$$\therefore \angle AEO = \angle ADB = 90^\circ, \text{ 即 } OC \perp AD,$$

$$\therefore AE = ED$$

(2) 解: $\therefore OC \perp AD$,

$$\therefore \widehat{AC} = \widehat{CD},$$

$$\therefore \angle ABC = \angle CBD = 36^\circ,$$

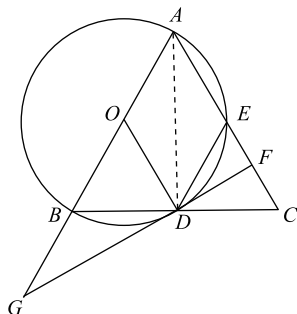
$$\therefore \angle AOC = 2\angle ABC = 2 \times 36^\circ = 72^\circ,$$

$$\therefore \widehat{AC} = \frac{72\pi \times 5}{180} = 2\pi,$$

$$S = \frac{72\pi \cdot 5^2}{360} = 5\pi.$$

21. (1) 证明: 连接 AD ,

$\therefore AB$ 为直径, $\therefore \angle ADB = 90^\circ, \therefore AD \perp BC$,



$$\therefore AB = AC, \therefore BD = CD,$$

$$\therefore AO = OB, \therefore OD = \frac{1}{2}AC, OD \parallel AC,$$

$$\therefore DF \text{ 为 } \odot O \text{ 的切线, } \therefore OD \perp DF, \therefore AC \perp DF,$$

$$\therefore \text{四边形 } ABDE \text{ 内接于圆 } O, \therefore \angle DEC = \angle ABD,$$

$$\therefore AB = AC, \therefore \angle ABD = \angle ACB, \therefore \angle DEC = \angle ACB,$$

$$\therefore DE = DC,$$

$$\therefore EF = CF;$$

$$(2) \text{Rt}\triangle ABD \text{ 中, } \cos \angle ABC = \frac{BD}{AB} = \frac{3}{5},$$

$$\therefore AB = 10, \therefore BD = 6, AC = 10,$$

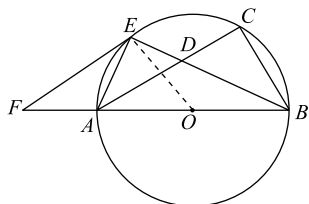
$$\therefore DC=BD=6, \quad S_{\triangle ACD}=\frac{1}{2}CD \cdot AD=\frac{1}{2}AC \cdot$$

$$DF, 10DF=6 \times 8, DF=\frac{24}{5},$$

$$\text{由勾股定理得: } AF=\sqrt{8^2-(\frac{24}{5})^2}=\frac{32}{5}.$$

22. (1) 证明: 连接 OE ,

$$\because \angle B \text{ 的平分线 } BE \text{ 交 } AC \text{ 于 } D, \therefore \angle CBE = \angle ABE.$$



$$\because EF \parallel AC, \therefore \angle CAE = \angle FEA.$$

$$\because \angle OBE = \angle OEB, \angle CBE = \angle CAE,$$

$$\therefore \angle FEA = \angle OEB.$$

$$\because \angle AEB = 90^\circ, \therefore \angle FEO = 90^\circ.$$

$\therefore EF$ 是 $\odot O$ 切线.

(2) 解: 在 $\triangle FEA$ 与 $\triangle FBE$ 中,

$$\because \angle F = \angle F, \angle FEA = \angle FBE,$$

$$\therefore \triangle FEA \sim \triangle FBE,$$

$$\therefore \frac{AF}{EF} = \frac{EF}{BF} = \frac{AE}{BE},$$

$$\therefore AF \cdot BF = EF \cdot EF,$$

$$\therefore AF \times (AF + 15) = 10 \times 10, \quad \text{解得 } AF = 5. \quad \therefore$$

$$BF = 20.$$

$$\therefore \frac{10}{20} = \frac{AE}{BE}, \quad \therefore BE = 2AE,$$

$$\because AB \text{ 为 } \odot O \text{ 的直径}, \therefore \angle AEB = 90^\circ,$$

$$\therefore AE^2 + BE^2 = 15^2,$$

$$\therefore AE^2 + (2AE)^2 = 225,$$

$$\therefore AE = 3\sqrt{5}.$$

测试卷(十)

一、选择题

1.D 2.A 3.D 4.B 5.C 6.B 7.D 8.B 9.B

10.B

二、填空题

11. 中心投影 12. 13 13. C 14. 70 15. SSS 16.

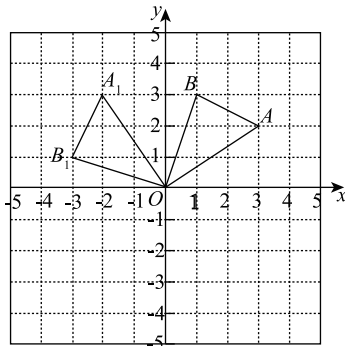
60 17. 6

三、解答题

18. 解: (1) 如图, $\triangle A_1OB_1$ 为所作;

$$(2) OA = \sqrt{2^2 + 3^2} = \sqrt{13},$$

$$\text{所以 } A_1 \text{ 旋转经过的路程长} = \frac{90 \cdot \pi \cdot \sqrt{13}}{180} =$$



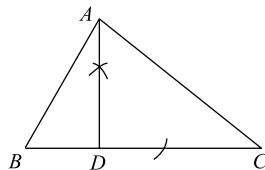
$$\frac{\sqrt{13}}{2}\pi.$$

19. 解: (1) 如图, AD 为所作.

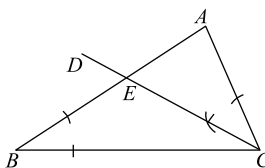
$$(2) \text{ 在 } \text{Rt} \triangle ABC \text{ 中}, BC = \sqrt{6^2 + 8^2} = 10,$$

$$\therefore \frac{1}{2}AD \cdot BC = \frac{1}{2}AB \cdot AC,$$

$$\therefore AD = \frac{6 \times 8}{10} = 4.8.$$



20. 解: (1) 如图, CD 为所作;



$$(2) \because \angle ACE = \angle ABC, \angle CAE = \angle BAC,$$

$$\therefore \triangle ACE \sim \triangle ABC,$$

$$\therefore AE:AC = AC:AB,$$

$$\text{即 } AE:6 = 6:8,$$

$$\therefore AE = \frac{9}{2}.$$

21. 解: $\because CD \parallel AB,$

$$\therefore \triangle EAB \sim \triangle ECD,$$

$$\therefore \frac{CD}{AB} = \frac{DE}{BE}, \text{ 即 } \frac{2}{AB} = \frac{3}{3+BD} \text{ ①},$$

$$\because FG \parallel AB,$$

$$\therefore \triangle HFG \sim \triangle HAB,$$

$$\therefore \frac{FG}{AB} = \frac{HG}{HB}, \text{ 即 } \frac{2}{AB} = \frac{5}{BD+5+5} \text{ ②},$$

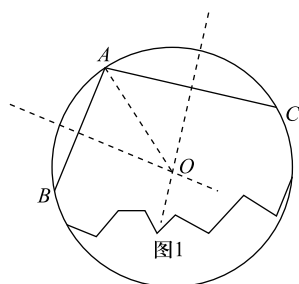
$$\text{由 ①② 得 } \frac{3}{3+BD} = \frac{5}{BD+5+5},$$

$$\text{解得 } BD = 7.5,$$

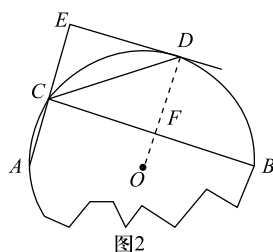
$$\therefore \frac{2}{AB} = \frac{3}{7.5+3}, \text{解得: } AB=7.$$

答:路灯杆 AB 的高度为 7 m.

22. (1) 解: 如图 1: 点 O 即为所求.



(2) ① 证明: 如图 2 中, 连接 OD 交 BC 于 F .



$\because AD$ 平分 $\angle BAC$,

$\therefore \angle DAC = \angle DAB$,

$\therefore \widehat{CD} = \widehat{BD}$,

$\therefore OD \perp BC$,

$\therefore CF = BF, \angle CFD = 90^\circ$,

$\because DE$ 是切线,

$\therefore DE \perp OD$,

$\therefore \angle EDF = 90^\circ$,

$\because AB$ 是直径,

$\therefore \angle ACB = \angle BCE = 90^\circ$,

\therefore 四边形 $DECF$ 是矩形,

$\therefore \angle E = 90^\circ$,

$\therefore AE \perp DE$.

② \because 四边形 $DECF$ 是矩形,

$\therefore DE = CF = BF = 3$,

在 $\text{Rt} \triangle ACB$ 中, $AB = \sqrt{2^2 + 6^2} = 2\sqrt{10}$,

\therefore 残缺圆的半圆面积 $= \frac{1}{2} \cdot \pi \cdot (\sqrt{10})^2 = 5\pi$.