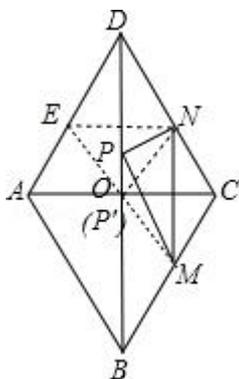


一. 选择题 (每小题 3 分共 36 分)

1. A. 2. C. 3. C. 4. B. 5. D. 6. A. 7. D. 8. A. 9. C. 10. B.

11. C. 解: 作点 N 关于 BD 的对称点 E , 连接 EM 交 BD 于 P' (与对角线的交点 O 重合). 此时 $\triangle P' MN$ 的周长最小.



\because 四边形 $ABCD$ 是菱形, $\therefore AC \perp BD, OA = OC = 3,$

在 $\text{Rt}\triangle DOC$ 中, $OD = \sqrt{CD^2 - OC^2} = \sqrt{5^2 - 3^2} = 4, \therefore BD = 2OD = 8, \therefore DN = NC, CM = BM,$

$\therefore MN = \frac{1}{2}BD = 4,$ 易知 $P'N = P'M = \frac{5}{2}, \therefore \triangle PMN$ 的周长的最小值为 9.

12. D. 解: ① $\because AE$ 平分 $\angle BAD, \therefore \angle BAE = \angle DAE, \because$ 四边形 $ABCD$ 是平行四边形,

$\therefore AD \parallel BC, \angle ABC = \angle ADC = 60^\circ, \therefore \angle DAE = \angle BEA, \therefore \angle BAE = \angle BEA, \therefore AB = BE = 1,$

$\therefore \triangle ABE$ 是等边三角形, $\therefore AE = BE = 1, \because BC = 2, \therefore EC = 1, \therefore AE = EC, \therefore \angle EAC = \angle ACE,$

$\because \angle AEB = \angle EAC + \angle ACE = 60^\circ, \therefore \angle ACE = 30^\circ, \because AD \parallel BC, \therefore \angle CAD = \angle ACE = 30^\circ, \text{故①正确;}$

② $\because BE = EC, OA = OC, \therefore OE = \frac{1}{2}AB = \frac{1}{2}, OE \parallel AB, \therefore \angle EOC = \angle BAC = 60^\circ + 30^\circ = 90^\circ,$

$\text{Rt}\triangle EOC$ 中, $OC = \sqrt{1^2 - (\frac{1}{2})^2} = \frac{\sqrt{3}}{2}, \because$ 四边形 $ABCD$ 是平行四边形, $\therefore \angle BCD = \angle BAD = 120^\circ,$

$\therefore \angle ACB = 30^\circ, \therefore \angle ACD = 90^\circ, \text{Rt}\triangle OCD$ 中, $OD = \sqrt{1^2 + (\frac{\sqrt{3}}{2})^2} = \frac{\sqrt{7}}{2}, \therefore BD = 2OD = \sqrt{7}, \text{故②正确;}$

③ 由②知: $\angle BAC = 90^\circ, \therefore S_{\text{平行四边形}ABCD} = AB \cdot AC, \text{故③正确;}$

④ 由②知: OE 是 $\triangle ABC$ 的中位线, $\therefore OE = \frac{1}{2}AB, \because AB = \frac{1}{2}BC, \therefore OE = \frac{1}{2}BC = \frac{1}{4}AD, \text{故④正确;}$

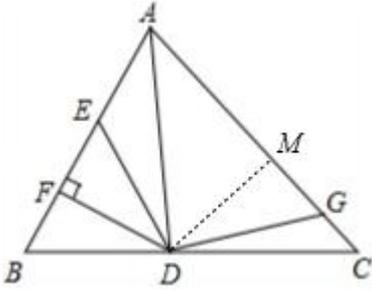
二. 填空题 (每小题 3 分共 18 分)

13. 八. 14. 2. 15. $\frac{24}{5}$. 16. 30.

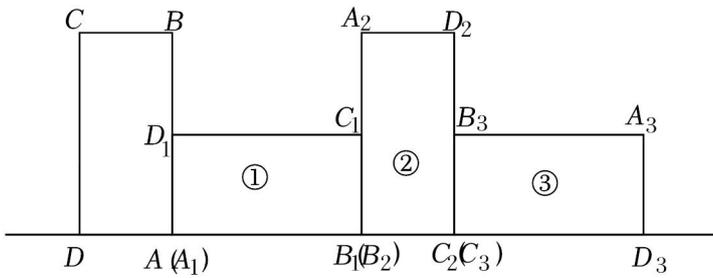
17. 41. 解: 作 $DM \perp AC$, 垂足为 M , 如图, $\because AD$ 是 $\triangle ABC$ 的角平分线, $DF \perp AB, DM \perp AC, \therefore DF = DM,$

$\because AD = AD, DF = DM, \therefore \triangle ADF \cong \triangle ADM (HL), \because DE = DG, DF = DM, \therefore \triangle DFE \cong \triangle DMG (HL),$

$\therefore S_{\triangle ADM} = S_{\triangle ADF} = S_{\triangle ADG} - S_{\triangle EFD} = 50 - 4.5 = 45.5, \therefore S_{\triangle AED} = S_{\triangle ADF} - S_{\triangle EFD} = 45.5 - 4.5 = 41.$



18. 3034.



解：∵ 四边形 $ABCD$ 是矩形，

$$\therefore AB = CD = 2 \text{ (cm)}, BC = AD = 1 \text{ (cm)},$$

第 1 次滚动后得到的长方形最右侧边与 CD 边的距离 $= 1 + 2 = 3 \text{ (cm)}$ ，

第 2 次滚动后得到的长方形最右侧边与 CD 边的距离 $= 1 + 2 + 1 = 4 \text{ (cm)}$ ，

第 3 次滚动后得到的长方形最右侧边与 CD 边的距离 $= 1 + 2 + 1 + 2 = 6 \text{ (cm)}$ ，

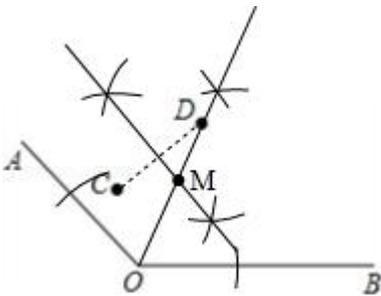
第 4 次滚动后得到的长方形最右侧边与 CD 边的距离 $= 1 + 2 + 1 + 2 + 1 = 7 \text{ (cm)}$ ，

...

第 2022 次滚动后得到的长方形最右侧边与 CD 边的距离 $= 1 + 3 \times \frac{2022}{2} = 3034 \text{ (cm)}$ ，

三. 解答题 (共 8 小题)

19. (6 分) 解：如图所示，点 M 即为所求。



20. (8 分) 解：(1) 设此多边形的边数为 n ，则 $(n-2) \cdot 180^\circ = 1620$ ，解得 $n = 11$ 。

故此多边形的边数为 11；

(2) 设多边形的一个内角为 $3x$ 度，则一个外角为 x 度，依题意得 $3x + x = 180$ ，解得 $x = 45$ 。

$360^\circ \div 45^\circ = 8$ 。故这个多边形的边数是 8。

21. (9分) 解: (1) $\because D$ 是 AB 的中点, E 是线段 AC 的中点, $\therefore DE \parallel BC, DE = \frac{1}{2}BC,$

$\because \angle ACB = 90^\circ, \therefore \angle DEC = 90^\circ, \because DF \perp DE, \therefore \angle EDF = 90^\circ, \therefore$ 四边形 $CEDF$ 是矩形,

$\therefore DE = CF = \frac{1}{2}BC, \therefore CF = BF = b, \because CE = AE = a, \therefore EF = \sqrt{CF^2 + CE^2} = \sqrt{a^2 + b^2};$

(2) $AE^2 + BF^2 = EF^2.$

证明: $\because BM \parallel AC,$ 则 $\angle AED = \angle BMD, \angle FBM = \angle ACB = 90^\circ,$

$\because D$ 点是 AB 的中点, $\therefore AD = BD,$ 在 $\triangle ADE$ 和 $\triangle BDM$ 中, $\begin{cases} \angle AED = \angle BMD \\ \angle ADE = \angle BDM, \\ AD = BD \end{cases} \therefore \triangle ADE \cong \triangle BDM (AAS),$

$\therefore AE = BM, DE = DM, \because DF \perp DE, \therefore EF = MF, \because$ 在 $Rt\triangle BMF$ 中 $BM^2 + BF^2 = MF^2, \therefore AE^2 + BF^2 = EF^2.$

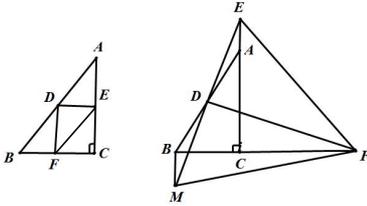


图 1

图 2

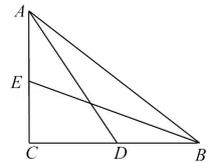
22. (8分) (1) 证明: \because 中线 $BE,$ 中线 $AD, CD = 4, CE = 3, \therefore AC = 6, BC = 8,$

$\because AB = 10, \therefore AB^2 = AC^2 + BC^2, \therefore \triangle ABC$ 是直角三角形, $\therefore \angle C = 90^\circ;$

(2) 解: $\because \angle C = 90^\circ, AD = 6, BE = 8, \therefore AC^2 + CD^2 = AD^2, BC^2 + CE^2 = BE^2,$

\because 中线 $BE,$ 中线 $AD, \therefore AC^2 + (\frac{1}{2}BC)^2 = 36, BC^2 + (\frac{1}{2}AC)^2 = 64,$

$\therefore \frac{5}{4}AC^2 + \frac{5}{4}BC^2 = 100, \therefore AC^2 + BC^2 = 80, \therefore AB = \sqrt{AC^2 + BC^2} = 4\sqrt{5}.$



23. (8分) 证明: (1) \because 四边形 $ABCD$ 是平行四边形, $\therefore AD \parallel BC, AD = BC, \therefore \angle D = \angle BCE,$

在 $\triangle ADC$ 和 $\triangle BCE$ 中,

$$\begin{cases} AD = BC \\ \angle D = \angle BCE, \\ CD = EC \end{cases}$$

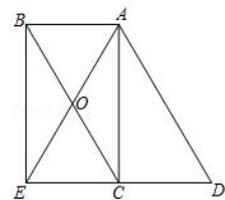
$\therefore \triangle ADC \cong \triangle BCE (SAS);$

(2) \because 四边形 $ABCD$ 是平行四边形, $\therefore AB = CD, AB \parallel CD,$

$\because CD = CE, \therefore AB \parallel CE, AB = CE, \therefore$ 四边形 $ABEC$ 是平行四边形,

$\therefore AE = 2OE, BC = 2OC,$ 又 $\because \angle BOE = 2\angle BCE, \angle BOE = \angle BCE + \angle OEC, \therefore \angle BCE = \angle OEC, \therefore OE = OC,$

$\therefore AE = BC, \therefore$ 平行四边形 $ABEC$ 是矩形.



24. (8分) 解: (1) 四边形 $CEGF$ 为菱形, 理由是: \because 四边形 $ABCD$ 是矩形, $\therefore AD \parallel BC, \therefore \angle GFE = \angle FEC,$

\because 图形翻折后点 G 与点 C 重合, EF 为折线, $\therefore \angle GEF = \angle FEC, \therefore \angle GFE = \angle FEG, \therefore GF = GE,$

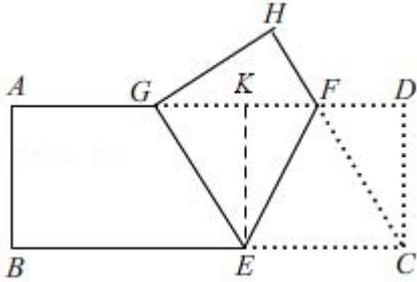
\because 图形翻折后 EC 与 GE 完全重合, $\therefore GE = EC, \therefore GF = EC, \therefore$ 四边形 $CEGF$ 为平行四边形,

\therefore 四边形 $CEGF$ 为菱形;

(2) 如图, 过 E 作 $EK \perp AD$ 于 K , 则 $EK = AB = 4$, (1) 得四边形 $CEGF$ 是菱形,

\therefore 四边形 $CEGF$ 的面积是 20, $\therefore FG \cdot EK = 20$, $4FG = 20$, $\therefore FG = 5$, $\therefore EG = 5$, $\therefore KG = \sqrt{5^2 - 4^2} = 3$,

$\therefore FK = 5 - 3 = 2$, $\text{Rt}\triangle EKF$ 中, $EF = \sqrt{EK^2 + FK^2} = \sqrt{4^2 + 2^2} = 2\sqrt{5}$.



25. (9分) (1) 解: 四边形 $BDFG$ 是平行四边形, 理由如下:

\therefore 将 $\triangle AED$ 剪下平移到 $\triangle BGC$, 将 $\triangle ABE$ 剪下平移到 $\triangle DCF$, $\therefore \triangle AED \cong \triangle BGC$, $\triangle ABE \cong \triangle DCF$,

$\therefore AE = BG$, $ED = GC$; $AE = DF$, $BE = CF$; $\therefore BG = DF$, $ED + BE = GC + CF$, 即 $BD = GF$,

\therefore 四边形 $BDFG$ 是平行四边形;

(2) 解: 若 $AE \perp BD$, 则 $\angle AED = 90^\circ$, $\therefore \angle G = \angle AED$, $\therefore \angle G = 90^\circ$, 由 (2) 知四边形 $BDFG$ 是平行四边形, 四边形 $BDFG$ 是矩形.

26. (10分) 解: (1) 如图 1, 将 $\triangle ADF$ 绕点 A 顺时针旋转, 使 AD 与 AB 重合, 得到 $\triangle ABF'$,

$\therefore \angle EAF = 45^\circ$, $\therefore \angle EAF' = \angle EAF = 45^\circ$, 在 $\triangle AEF$ 和 $\triangle AEF'$ 中,

$$\begin{cases} AF = AF' \\ \angle EAF' = \angle EAF, \therefore \triangle AEF \cong \triangle AEF' \quad (SAS), \\ AE = AE \end{cases}$$

$\therefore EF = EF'$, 又 $EF' = BE + BF' = BE + DF$, $\therefore EF = BE + DF$;

(2) 结论 $EF = BE + DF$ 仍然成立.

理由如下: 如图 2, 将 $\triangle ADF$ 绕点 A 顺时针旋转, 使 AD 与 AB 重合,

得到 $\triangle ABF'$, 则 $\triangle ADF \cong \triangle ABF'$,

$\therefore \angle BAF' = \angle DAF$, $AF' = AF$, $BF' = DF$, $\angle ABF' = \angle D$, 又 $\therefore \angle EAF = \frac{1}{2} \angle BAD$,

$\therefore \angle EAF = \angle DAF + \angle BAE = \angle BAE + \angle BAF'$, $\therefore \angle EAF = \angle EAF'$,

又 $\therefore \angle ABC + \angle D = 180^\circ$, $\therefore \angle ABF' + \angle ABE = 180^\circ$, $\therefore F'$ 、 B 、 E 三点共线, 在 $\triangle AEF$ 与 $\triangle AEF'$ 中,

$$\begin{cases} AF = AF' \\ \angle EAF = \angle EAF', \therefore \triangle AEF \cong \triangle AEF' \quad (SAS), \therefore EF = EF', \text{ 又 } \therefore EF' = BE + BF', \therefore EF = BE + DF; \\ AE = AE \end{cases}$$

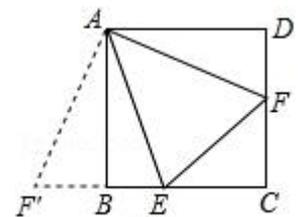


图1

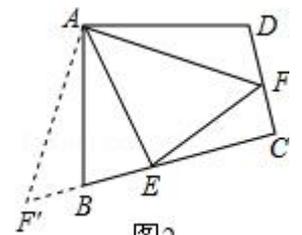


图2

(3) 发生变化. EF 、 BE 、 DF 之间的关系是 $EF = BE - DF$.

理由如下: 如图 3, 将 $\triangle ADF$ 绕点 A 顺时针旋转, 使 AD 与 AB 重合, 点 F 落在 BC 上点 F' 处, 得到 $\triangle ABF'$,

$\therefore \triangle ADF \cong \triangle ABF'$, $\therefore \angle BAF' = \angle DAF$, $AF' = AF$, $BF' = DF$,

又 $\because \angle EAF = \frac{1}{2} \angle BAD$, 且 $\angle BAF' = \angle DAF$,

$\therefore \angle F'AE = \angle BAD - (\angle BAF' + \angle EAD) = \angle BAD - (\angle DAF + \angle EAD) = \angle BAD - \angle FAE = \angle FAE$,

即 $\angle F'AE = \angle FAE$, 在 $\triangle F'AE$ 与 $\triangle FAE$ 中,
$$\begin{cases} AF' = AF \\ \angle F'AE = \angle FAE \\ AE = AE \end{cases} \therefore \triangle F'AE \cong \triangle FAE \text{ (SAS)},$$

$\therefore EF = EF'$, 又 $\because BE = BF' + EF'$, $\therefore EF' = BE - BF'$, 即 $EF = BE - DF$.

