

2021 学年九年级第二学期模拟练习数学学科

答案要点及评分标准

一、选择题：

1. C; 2. B; 3. B; 4. C; 5. A; 6. D.

二、填空题：

7. $2x(x-3)$; 8. $16\vec{a}+12\vec{b}$; 9. $\frac{\sqrt{3}}{2}$; 10. $x=-23$; 11. $-3<x<4$; 12. $\frac{1}{3}$;

13. 15; 14. 36; 15. $(\frac{\sqrt{3}}{3}, \sqrt{3})$; 16. $3\sqrt{5}$; 17. 4; 18. $\sqrt{2}$.

三、解答题：

19. 解：原式 $= \frac{1}{3} + 4 - \sqrt{3} - 3 + 2 + \sqrt{3} \dots\dots\dots (2 \text{ 分} + 2 \text{ 分} + 2 \text{ 分} + 2 \text{ 分})$
 $= 3\frac{1}{3} \dots\dots\dots (2 \text{ 分})$

20. 解：由②得： $2x+3y=0$, $2x-3y=0 \dots\dots\dots (2 \text{ 分})$

原方程组可化为 $\begin{cases} x+y=5 \\ 2x+3y=0 \end{cases}$, $\begin{cases} x+y=5 \\ 2x-3y=0 \end{cases} \dots\dots\dots (2 \text{ 分})$

解得原方程组的解为 $\begin{cases} x_1=15 \\ y_1=-10 \end{cases}$, $\begin{cases} x_2=2 \\ y_2=3 \end{cases} \dots\dots\dots (4 \text{ 分})$

\therefore 原方程组的解是 $\begin{cases} x_1=15 \\ y_1=-10 \end{cases}$, $\begin{cases} x_2=2 \\ y_2=3 \end{cases} \dots\dots\dots (2 \text{ 分})$

21. 解：设玩具厂改良生产线前每天生产 x 箱“冰墩墩”. $\dots\dots\dots (1 \text{ 分})$

根据题意，列方程得 $\frac{6000}{x} = \frac{6000}{x+20} - 10$; $\dots\dots\dots (2 \text{ 分})$

化简得： $x^2 + 20x - 12000 = 0$; $\dots\dots\dots (2 \text{ 分})$

解得： $x_1=100$, $x_2=-120$; $\dots\dots\dots (2 \text{ 分})$

经检验， $x_1=100$ 是原方程的根，且符合题意， $x_2=-120$ 不符合题意舍去. (2 分)

答：玩具厂改良生产线前每天生产 100 箱“冰墩墩”. $\dots\dots\dots (1 \text{ 分})$

22. 解：(1) B. $\dots\dots\dots (2 \text{ 分})$

(2) $0 < \text{pre}A < 2$. $\dots\dots\dots (2 \text{ 分})$

(3) 过点 B 作 $BD \perp AC$, 垂足为点 D (1 分)

$$\because \sin A = \frac{8}{17}, \therefore \text{令 } AB=17k, BD=8k, (k \neq 0) \dots\dots\dots (1 \text{ 分})$$

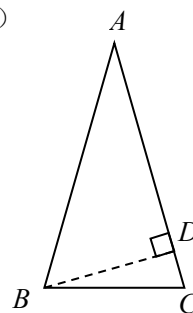
在 $\text{Rt}\triangle ABD$ 中, $\angle ADB=90^\circ$,

$$AD = \sqrt{AB^2 - BD^2} = \sqrt{(17k)^2 - (8k)^2} = 15k, \dots\dots\dots (1 \text{ 分})$$

\because 等腰 $\triangle ABC$, $\therefore AB=AC=17k$. $\therefore DC=2k$ (1 分)

$$\text{在 } \text{Rt}\triangle BCD \text{ 中}, BC = \sqrt{(8k)^2 + (2k)^2} = 2\sqrt{17}k. \dots (1 \text{ 分})$$

$$\therefore \cos A = \frac{BC}{AB} = \frac{2\sqrt{17}k}{17k} = \frac{2\sqrt{17}}{17}. \dots\dots\dots (1 \text{ 分})$$



(第 22 题图)

23. 证明: (1) \because 四边形 $ABCD$ 是矩形, $\therefore \angle B = \angle ECD = 90^\circ$;

$$\therefore \angle BAE + \angle BEA = 90^\circ. \dots\dots\dots (1 \text{ 分})$$

又 $\because FG \perp BC$, $\therefore \angle BGF = \angle B = 90^\circ$; (1 分)

\because 线段 AE 绕点 E 顺时针旋转 90° , 即: $\angle AEF = 90^\circ$,

$$\therefore \angle GEF + \angle BEA = 90^\circ; \dots\dots\dots (1 \text{ 分})$$

$$\therefore \angle BAE = \angle GEF. \dots\dots\dots (1 \text{ 分})$$

在 $\triangle ABE$ 与 $\triangle EGF$ 中,

$$\begin{cases} \angle B = \angle BGF \\ \angle BAE = \angle GEF \\ AE = FE \end{cases}$$

$$\therefore \triangle ABE \cong \triangle EGF. \dots\dots\dots (1 \text{ 分})$$

$$\therefore BE = FG. \dots\dots\dots (1 \text{ 分})$$

(2) $\because \angle B = \angle ECD$, $\angle BAE = \angle GEF$,

$$\therefore \triangle ABE \sim \triangle ECM. \therefore \frac{AB}{EC} = \frac{AE}{EM}. \dots\dots\dots (1 \text{ 分})$$

$$\because AB \cdot DM = EC \cdot AE, \therefore \frac{AB}{EC} = \frac{AE}{DM}. \dots\dots\dots (1 \text{ 分})$$

$$\therefore \frac{AE}{EM} = \frac{AE}{DM}. \therefore EM = DM. \dots\dots\dots (1 \text{ 分})$$

在 $\text{Rt}\triangle AEM$ 与 $\text{Rt}\triangle ADM$ 中, $\angle AEF = \angle D = 90^\circ$

$$\begin{cases} EM = DM \\ AM = AM \end{cases}$$

$$\therefore \text{Rt}\triangle AEM \cong \text{Rt}\triangle ADM. \dots\dots\dots (1 \text{ 分})$$

$$\therefore AD = AE.$$

\therefore 点 A 在线段 DE 的垂直平分线上; (1 分)

$\because EM = DM$, \therefore 点 M 在线段 DE 的垂直平分线上. (1 分)

$\therefore AM$ 垂直平分 DE .

24. 解: (1) \because 点 A 坐标为 $(-1, 0)$, 点 B 坐标为 $(3, 0)$.

设抛物线 $y = a(x+1)(x-3) (a \neq 0)$,

\because 抛物线经过点 $C(0, 4)$,

$\therefore 4 = -3a$. 解得 $a = -\frac{4}{3}$ (2 分)

\therefore 抛物线的表达式是 $y = -\frac{4}{3}x^2 + \frac{8}{3}x + 4$ (2 分)

(2) ① 由于 $\odot G$ 与 $\odot E$ 内切,

当 $r_{\odot G} > r_{\odot E}$ 时, 则 $GB - EF = GE$ (1 分)

又 $\because GE = GB - EB$, $\therefore EF = EB$ (1 分)

当 $r_{\odot G} < r_{\odot E}$ 时, 则 $EF - GB = GE$ (1 分)

设 $EF = 5t$, $FG = 3t$, $GE = 4t$, 则 $5t - GB = 4t$,

$\therefore GB = t < GE = 4t$, \therefore 点 E 在线段 CB 的延长线上.

又 \because 已知点 E 在线段 BC 上, \therefore 矛盾, 因此不存在.

② $\because OC \perp OB$, $FD \perp OB$, $\therefore \angle COB = \angle EDB = 90^\circ$.

$\therefore \tan \angle OBC = \frac{ED}{BD} = \frac{OC}{OB} = \frac{4}{3}$. \therefore 设 $BD = t$, 则 $DE = \frac{4}{3}t$ (1 分)

\therefore 在 $Rt\triangle BED$ 中, $\angle BDE = 90^\circ$, $BE = \sqrt{DE^2 + DB^2} = \sqrt{t^2 + (\frac{4}{3}t)^2} = \frac{5}{3}t$.

$\therefore DF = DE + EF = \frac{4}{3}t + \frac{5}{3}t = 3t$ (1 分)

$\therefore F$ 坐标为 $(3-t, 3t)$ (1 分)

$\because F$ 点在抛物线 $y = -\frac{4}{3}x^2 + \frac{8}{3}x + 4$ 上,

$\therefore 3t = -\frac{4}{3}(3-t)^2 + \frac{8}{3}(3-t) + 4$ (1 分)

\therefore 解得 $t = \frac{7}{4}$, $t = 0$ (点 F 与点 B 重合, 舍去).

$\therefore F$ 坐标为 $(\frac{5}{4}, \frac{21}{4})$ (1 分)

25. (1) 解: 过点 A 作 $AH \perp BC$, 垂足为点 G ,

$$\because \text{在 Rt}\triangle ABH \text{ 中, } \angle AHB=90^\circ, AB=26, \cos B = \frac{5}{13},$$

$$\therefore BH = 10, AH = 24. \dots\dots\dots (1 \text{ 分})$$

$$\therefore CH = BC - BH = 32.$$

$$\because \text{在 Rt}\triangle AHC \text{ 中, } \angle AHC=90^\circ, AH = 24, CH = 32,$$

$$\therefore AC = \sqrt{AH^2 + CH^2} = 40. \dots\dots\dots (1 \text{ 分})$$

过点 D 作 $DH \perp AC$, 垂足为点 E ,

$$\because AD = DC, \therefore \angle DAC = \angle DCA, AE = \frac{1}{2} AC = 20.$$

$$\because AD \parallel BC, \therefore \angle DAC = \angle ACB = \angle DCA \dots\dots\dots (1 \text{ 分})$$

$$\therefore \text{在 Rt}\triangle ADE \text{ 中, } \cos \angle DAC = \frac{AE}{AD} = \cos \angle ACB = \frac{CH}{AC} = \frac{4}{5},$$

$$\therefore AD = CD = 25. \dots\dots\dots (1 \text{ 分})$$

(2) 参考方法一: 以 C 为圆心, CM 为半径作圆, 交射线 CD 于点 N , 联结 MN .

$$\because CM = CN, \angle ACB = \angle DCA,$$

$$\therefore CG \perp AC, MG = \frac{1}{2} MN. \dots\dots\dots (1 \text{ 分})$$

$$\because \text{在 Rt}\triangle MGC \text{ 中, } \angle MGC=90^\circ, CM=x, \sin \angle ACB = \frac{3}{5},$$

$$\therefore MG = \frac{3}{5}x, CG = \frac{4}{5}x, \therefore AG = AC - GC = \frac{4}{5}x. \dots\dots\dots (1 \text{ 分})$$

$$\therefore y = \frac{AG}{GC} = \frac{50-x}{x} \quad (0 < x \leq 25). \dots\dots\dots (1 \text{ 分}+1 \text{ 分})$$

参考方法二: 延长 MN 交 AD 的延长线于点 F .

$$\because AD \parallel BC, \therefore \frac{DF}{CM} = \frac{DN}{CN}, \frac{AF}{CM} = \frac{AG}{GC} \dots\dots\dots (1 \text{ 分})$$

$$\because CM = CN = x, CD = AD = 25, \therefore DN = 25 - x,$$

$$\therefore \frac{DF}{x} = \frac{25-x}{x}, \therefore DF = 25 - x \dots\dots\dots (1 \text{ 分})$$

$$\therefore AF = 25 - x,$$

$$\therefore y = \frac{AG}{GC} = \frac{50-x}{x} \quad (0 < x \leq 25). \dots\dots\dots (1 \text{ 分}+1 \text{ 分})$$

(3) 当点 N 在线段 CD 上时,

$$\because CM = CN, \therefore \angle NMC = \angle MNC.$$

$$\because \angle NMC = 2\angle DMN, \angle MNC = \angle DMN + \angle MDN,$$

$$\therefore \angle DMN = \angle MDN.$$

$$\therefore DN = MN = 25 - x \dots\dots\dots (1 \text{ 分})$$

$$\because MG = \frac{3}{5}x, \quad MG = \frac{1}{2}MN, \quad \therefore MN = \frac{6}{5}x.$$

$$\therefore \frac{6}{5}x = 25 - x \dots\dots\dots (1 \text{ 分})$$

$$\therefore x = \frac{125}{11}, \text{ 即 } CM = \frac{125}{11}. \dots\dots\dots (1 \text{ 分})$$

当点 N 在线段 CD 的延长线上时, $DN = x - 25$.

延长 DA 交射线 MN 于点 P .

$$\because \angle NMC = 2\angle DMN, \quad \therefore \angle NMD = \angle DMC \dots\dots\dots (1 \text{ 分})$$

$$\because AD \parallel BC, \quad \angle NMC = \angle MNC, \quad \therefore \angle NGD = \angle MNC, \quad \frac{NG}{GM} = \frac{DN}{DC}.$$

$$\therefore DN = GD = x - 25.$$

$$\because AD \parallel BC, \quad \therefore \angle GDM = \angle DMC, \quad \therefore \angle NMD = \angle GDM.$$

$$\therefore GM = GD = x - 25.$$

$$\therefore \frac{\frac{6}{5}x - (x - 25)}{x - 25} = \frac{x - 25}{25}, \dots\dots\dots (1 \text{ 分})$$

$$\therefore x = 55, \text{ 即 } CM = 55. \dots\dots\dots (1 \text{ 分})$$

综上所述, 线段 CM 的长为 $\frac{125}{11}$ 或 55 .