

2022 年秋九年级学科核心素养质量监测数学试题

参 考 答 案

一、选择题（本大题共 10 小题，每小题 4 分，共 40 分。）

1. C 2. A 3. B 4. A 5. C 6. D 7. B 8. B 9. A 10. D

二、填空题（本大题共 6 小题，每小题 4 分，共 24 分）

11. ≥ 1 12. $k < 1$ 13. $\frac{3}{5}$ 14. 9 15. 33 16. ①②④

三、解答题（本大题共 9 小题，共 86 分。解答应写出文字说明，证明过程或演算步骤）

17 解：(1) $\sqrt{4} + \sqrt{8} - \sqrt{2}$

$$= 2 + 2\sqrt{2} - \sqrt{2} \text{-----} 4$$

$$= 2 + \sqrt{2} \text{-----} 8$$

18 解：(1) $(x - 5)^2 = 5 - x$,

$$(x - 5)^2 + (x - 5) = 0 \text{ 或 } x^2 - 9x + 20 = 0 \text{-----} 2$$

$$(x - 5)(x - 5 + 1) = 0 \text{ 或 } (x - 5)(x - 4) = 0, \text{-----} 4$$

$$x - 5 = 0 \text{ 或 } x - 4 = 0 \text{-----} 6$$

$$x_1 = 5, x_2 = 4. \text{ (或用求根公式) } \text{-----} 8$$

19. 解：(1) 证明：

$$\because DE \parallel AC, \therefore \angle BED = \angle C, \text{-----} 4$$

$$\because EF \parallel AB, \therefore \angle B = \angle FEC,$$

$$\therefore \triangle BDE \sim \triangle EFC; \text{-----} 8$$

(也可用相似的传递性)

20. 解：(1) $A = \left(\frac{m}{n} - \frac{n}{m}\right) \cdot \frac{\sqrt{3}mn}{m-n}$

$$= \frac{m^2 - n^2}{mn} \cdot \frac{\sqrt{3}mn}{m-n} \text{-----} 2$$

$$= \frac{(m+n)(m-n)}{mn} \cdot \frac{\sqrt{3}mn}{m-n} \text{-----} 3$$

$$= \sqrt{3}(m+n); \text{-----} 4$$

$$(2) \because m+n-3\sqrt{3}=0, \quad \therefore m+n=3\sqrt{3}, \text{-----} 6$$

$$\text{当 } m+n=3\sqrt{3} \text{ 时, } A=\sqrt{3} \times 3\sqrt{3}=9. \text{-----} 8$$

21. 解: (1) 证明:

$$\because AD \text{ 平分 } \angle BAC, \therefore \angle EAC = \angle BAD, \text{-----} 2$$

$$\text{又 } \angle C = \angle D, \therefore \triangle ABD \sim \triangle AEC. \text{-----} 3$$

$$(2) \because AC \parallel BD, \therefore \angle CAE = \angle D, \text{-----} 4$$

$$\because \angle C = \angle D, \therefore \angle CAE = \angle C. \text{-----} 5$$

$$\because \triangle ACE \text{ 是等腰三角形, } \therefore AE = CE = 4, \text{-----} 6$$

$$\because \triangle ABD \sim \triangle AEC, \therefore \frac{AB}{AE} = \frac{AD}{AC}. \text{-----} 7$$

$$\text{故 } \frac{5}{4} = \frac{AD}{6}. \therefore AD = \frac{15}{2}. \text{-----} 8$$

$$22. \text{ 解: (1) } 100-x, \quad 300+2x, \quad 400-2x \text{ (每格 2 分)} \text{-----} 6$$

依题意得:

$$100 \times 300 + (100-x)(300+2x) + 50(400-2x) - 60 \times 1000 = 15200, \text{-----} 7$$

$$\text{整理得: } x^2 + 100x - 2400 = 0, \text{-----} 8$$

$$\text{解得: } x_1 = 20, x_2 = -120 \text{ (不合题意, 舍去)} \text{-----} 9$$

当 $x = 20$ 时, 售价为: $100 - x = 100 - 20 = 80 > 60$, 符合题意.

答: 十月份的销售单价应是 80 元. ----- 10

$$23. \text{ 解: (1) } \because a = 1, b = -(m-2), c = -\frac{m^2}{4},$$

$$\therefore \Delta = b^2 - 4ac = [-(m-2)]^2 - 4 \times 1 \times \left(-\frac{m^2}{4}\right) \text{-----} 2$$

$$= 2m^2 - 4m + 4 = 2(m-1)^2 + 2 > 0,$$

$$\therefore \angle BAM = \angle DAN, \text{-----}2$$

$$\text{在} \triangle ABM \text{和} \triangle ADN \text{中}, \begin{cases} AB = AD \\ \angle BAM = \angle DAN \\ AM = AN \end{cases}$$

$$\therefore \triangle ABM \cong \triangle ADN(SAS); \text{-----}4$$

(2)证明: $\because AM = AN, \angle MAN = 90^\circ,$

$$\therefore \angle ANE = 45^\circ;$$

\because 四边形ABCD是正方形,

$$\therefore \angle ACB = 45^\circ = \angle ANE, \text{-----}5$$

又 \because AM平分 $\angle BAC$,

$$\therefore \angle CAM = \angle BAM,$$

$$\because \angle BAM = \angle DAN,$$

$$\therefore \angle CAM = \angle NAD,$$

$$\therefore \triangle AMC \sim \triangle AEN, \text{-----}6$$

$$\therefore \frac{AM}{AE} = \frac{AC}{AN},$$

$$\therefore AM \cdot AN = AC \cdot AE, \text{-----}7$$

$$\because AM = AN,$$

$$\therefore AM^2 = AC \cdot AN; \text{-----}8$$

$$(3) \because CM = (n-1)BM,$$

$$\therefore \text{设} BM = a, CM = (n-1)a,$$

$$\because \triangle ABM \cong \triangle ADN,$$

$$\therefore DN = BM = a, \angle ADN = \angle B,$$

\because 四边形ABCD是正方形,

$$\therefore \angle B = \angle ADC = \angle BCD = 90^\circ,$$

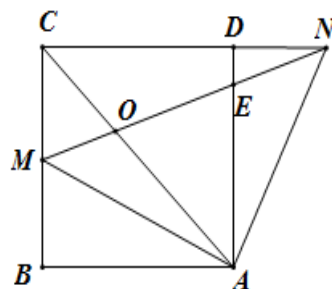
$$AB = BC = CD = AD = a + (n-1)a = na, \text{-----}9$$

$$AD \parallel BC, \therefore \angle ADC + \angle ADN = 180^\circ,$$

$\therefore C, D, N$ 三点共线,

$$\therefore CN = na + a = (n+1)a, \text{-----}10$$

$$\because AD \parallel BC, \therefore \frac{DN}{CN} = \frac{DE}{CM}, \frac{OM}{OE} = \frac{CM}{AE},$$



$$\therefore \frac{a}{(n+1)a} = \frac{DE}{(n-1)a}, \quad \therefore DE = \frac{n-1}{n+1} a, \quad \text{-----11}$$

$$\therefore AE = na - \frac{n-1}{n+1} a = (n^2 + 1)a/(n + 1), \quad \text{-----12}$$

$$\therefore \frac{OM}{OE} = \frac{CM}{AE} = (n-1)a/[(n^2 + 1)a/(n + 1)] = (n^2 - 1)/(n^2 + 1). \quad \text{-----13}$$

25: (1) 证明: $\because OA=OB$, P 为 AB 的中点,

$$\therefore OP \perp AB, OP=BP, \angle PON = \angle B = 45^\circ \quad \text{-----1}$$

$$\because PM \perp PN, \therefore \angle OPN + \angle OPM = 90^\circ, \quad \text{-----2}$$

$$\text{又 } \angle BPM + \angle OPM = 90^\circ \quad \therefore \angle OPN = \angle BPM \quad \text{-----3}$$

$$\therefore \triangle OPN \cong \triangle BPM \quad \therefore PM=PN \quad \text{-----4}$$

2) 解① $\because OM = x$, 且 $\triangle OPN \cong \triangle BPM$

$$\therefore ON = BM = 8-x \quad \text{-----5}$$

$$S_{\triangle PMN} = \frac{1}{2} PM \cdot PN = \frac{1}{2} PM^2 = \frac{1}{4} MN^2 = OM^2 + ON^2$$

$$\therefore y = \frac{1}{4} (x^2 + x^2 - 16x + 64) \quad \text{-----6}$$

$$= \frac{1}{2} (x^2 - 8x) + 16$$

$$= \frac{1}{2} (x-4)^2 + 8 \quad \text{-----7}$$

$$\therefore \text{当 } x=4 \text{ 时, } y \text{ 有最小值 } 8. \quad \text{-----8}$$

或(整理得: $x^2 - 8x + 16 - 32y = 0$, 方程有解, 得 $\Delta \geq 0$, $\therefore y \geq 8$)

(另解: 当 $PM \perp OB$ 时, PM 最小值为 4, 得面积的最小值亦给分!)

$$\textcircled{2} \because PM=PN, \therefore \angle PMN = 45^\circ, \text{ 又 } \angle DOM = \angle B = 45^\circ$$

$$\therefore \angle OMD + \angle PMB = 135^\circ, \text{ 又 } \angle BPM + \angle PMB = 135^\circ$$

(或 $\angle PMO = \angle PMN + \angle OMN = \angle MPB + \angle OBP$)

$$\therefore \angle OMD = \angle BPM, \therefore \triangle OMD \sim \triangle BPM \quad \text{-----9}$$

$$\therefore \frac{DM}{OM} = \frac{MP}{BP}, BP = \frac{1}{2} AB = \frac{1}{2} \sqrt{8^2 + 8^2} = 4\sqrt{2}$$

$$\therefore \frac{MP}{4\sqrt{2}} = \frac{\sqrt{10}}{4}, \text{ 得 } MP = 2\sqrt{5} \quad \text{-----10}$$

$$\therefore MN = \sqrt{2} MP = 2\sqrt{10} \quad \text{-----11}$$

在 $Rt\triangle OMN$ 中: $OM^2 + ON^2 = MN^2$

