

第 1 页 共 5 页

$$\because k > \frac{3}{4} \quad \therefore k = 2 \quad \dots\dots\dots 7 \text{ 分}$$

$$\therefore x_1 + x_2 = -(2k + 1) = -5 \quad \dots\dots\dots 8 \text{ 分}$$

20. (本小题 8 分)

解: (1) 向下; 直线 $x = 2$; $(2, 1)$; $(0, -3)$ (每空 1 分, 共 4 分) $\dots\dots\dots 4 \text{ 分}$

(2) $1 < x < 3$; $\dots\dots\dots 6 \text{ 分}$

(3) $-15 < y \leq 1$; $\dots\dots\dots 8 \text{ 分}$

21. (本小题 8 分)

(1) ① $60 - 40 - x$; (也可以写成 “ $20 - x$ ”) $\dots\dots\dots 1 \text{ 分}$

② $300 + 30x$ $\dots\dots\dots 2 \text{ 分}$

③ $y = (60 - 40 - x)(300 + 30x)$ $\dots\dots\dots 3 \text{ 分}$

(2) 解: $y = (60 - 40 - x)(300 + 30x)$

$$= -30x^2 + 300x + 6000 \quad \dots\dots\dots 4 \text{ 分}$$

$$\because a = -30 < 0$$

\therefore 抛物线的开口方向向下

$$\text{令 } x = -\frac{300}{2 \times (-30)} = 5 \quad \dots\dots\dots 5 \text{ 分}$$

$$\therefore \begin{cases} 20 - x \geq 0 \\ x \geq 0 \end{cases} \text{ 得: } 0 \leq x \leq 20 \quad \dots\dots\dots 6 \text{ 分}$$

$$\therefore \text{当 } x = 5 \text{ 时, } y_{\max} = 11250 \quad \dots\dots\dots 7 \text{ 分}$$

因此, 当定价为 $60 - 5 = 55$ 元时, 能使每星期的利润最大, 其最大值是 11250 元. $\dots\dots\dots 8 \text{ 分}$

22. (本小题 10 分)

解: (1) ① 如图 1, 由题意得 $A(2, 2)$ 是上边缘抛物线的顶点,

$$\text{设 } y = a(x - 2)^2 + 2 \quad (a \neq 0),$$

又 \because 抛物线过点 $(0, 1.5)$, $\therefore 1.5 = 4a + 2$, $\dots\dots\dots 1 \text{ 分}$

$$\therefore a = -\frac{1}{8},$$

\therefore 上边缘抛物线的函数解析式为 $y = -\frac{1}{8}(x - 2)^2 + 2$, $\dots\dots\dots 2 \text{ 分}$

当 $y = 0$ 时, $-\frac{1}{8}(x - 2)^2 + 2 = 0$, $\dots\dots\dots 3 \text{ 分}$

解得: $x_1 = 6$, $x_2 = -2$ (舍去),

\therefore 喷出水的最大射程 OC 为 6 m ; $\dots\dots\dots 4 \text{ 分}$

(2) \because 对称轴为直线 $x = 2$,

\therefore 点 $(0, 1.5)$ 的对称点为 $(4, 1.5)$, $\dots\dots\dots 5 \text{ 分}$

∴ 下边缘抛物线是由上边缘抛物线向左平移 $4m$ 得到的,6 分

∴ 点 B 的坐标为 $(2, 0)$;7 分

(3) $2 \leq d \leq 2\sqrt{3} - 1$ 10 分

23. (本小题 10 分)

证明: (1) ∵ $\angle BAC = \angle DAE$

$$\therefore \angle BAD + \angle DAC = \angle DAC + \angle CAE$$

$$\therefore \angle BAD = \angle CAE \dots\dots\dots 1 \text{ 分}$$

$$\therefore AB = AC, AD = AE$$

$$\therefore \triangle BAD \cong \triangle CAE \dots\dots\dots 3 \text{ 分}$$

(2) 连接 EC , 如图 1.

$$\therefore \angle BAC = \angle DAE$$

$$\therefore \angle BAD + \angle DAC = \angle DAC + \angle CAE$$

$$\therefore \angle BAD = \angle CAE$$

$$\therefore AB = AC, AD = AE$$

$$\therefore \triangle BAD \cong \triangle CAE \dots\dots\dots 4 \text{ 分}$$

$$\therefore BD = CE, \angle B = \angle ACE$$

$$\therefore AB = AC, \angle BAC = 90^\circ$$

$$\therefore \angle B = \angle ACB = 45^\circ = \angle ACE$$

$$\therefore \angle DCE = 90^\circ$$

$$\therefore CD^2 + CE^2 = DE^2 \dots\dots\dots 5 \text{ 分}$$

$$\therefore AD = AE, \angle DAE = 90^\circ$$

$$\therefore DE^2 = AD^2 + AE^2 = 2AD^2$$

$$\therefore CD^2 + CE^2 = 2AD^2$$

$$\therefore BD = CE$$

$$\therefore BD^2 + CD^2 = 2AD^2 \dots\dots\dots 6 \text{ 分}$$

(3) 延长 DE 至 F , 使得 $EF = ED$, 连接 FA, FB . 如图 2.

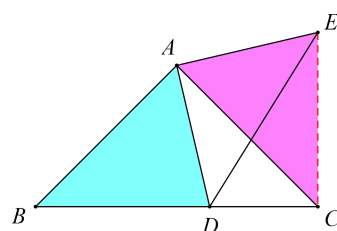
$$\therefore EF = ED = EA, AE \perp FD$$

$$\therefore AF = AD, \angle AFE = \angle ADE = 45^\circ$$

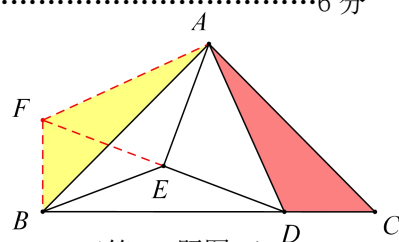
$$\therefore \angle FAD = 90^\circ$$

$$\therefore \angle BAC = \angle FAD = 90^\circ \dots\dots\dots 6 \text{ 分}$$

$$\therefore \angle FAB + \angle BAD = \angle BAD + \angle DAC$$



(第 23 题图 1)



(第 23 题图 2)

$\therefore \angle FAB = \angle DAC$
 $\because AB = AC, AF = AD$
 $\therefore \triangle AFB \cong \triangle ADC$ 8 分
 $\therefore \angle ABF = \angle C$
 $\because AB = AC, \angle BAC = 90^\circ$
 $\therefore \angle ABC = \angle C = 45^\circ$
 $\therefore \angle FBD = 90^\circ$ 9 分
 $\because EF = ED$
 $\therefore EB = ED$ 10 分

24. (本小题 12 分)

(1) ① 将 $(-2, 3)$ 和 $(1, 0)$ 分别代入抛物线解析式中

$$\begin{cases} 4a + 4 + c = 3 \\ a - 2 + c = 0 \end{cases} \text{1 分}$$

$$\text{解得: } \begin{cases} a = -1 \\ c = 3 \end{cases} \text{2 分}$$

$$\therefore \text{抛物线的解析式为 } -x^2 - 2x + 3 \text{3 分}$$

② 本题分两种情况:

(I) 当 P 点在第一象限时, (如图 1)

连接 AP 交 OC 于 E 点

$$\text{令 } y = 0, \text{ 得: } x_A = -3, x_B = 1,$$

$$\therefore OA = 3, OB = 1$$

$$\text{令 } x = 0, \text{ 得: } y_C = 3,$$

$$\therefore OC = 3 = OA$$

$$\because \angle PAB = \angle BCO, \angle BOC = \angle EOA = 90^\circ$$

$$\therefore \triangle BOC \cong \triangle EOA$$

$$\therefore OE = OB = 1$$

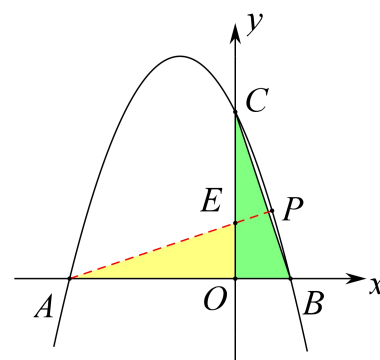
$$\therefore E \text{ 点坐标为 } (0, 1) \text{4 分}$$

设直线 AE 的解析式为 $y = kx + 1 (k \neq 0)$

将点 $A(-3, 0)$ 代入, 得到: $-3k + 1 = 0$

$$\text{解得: } k = \frac{1}{3}, \text{ 即: 直线 } AE \text{ 的解析式为 } y = \frac{1}{3}x + 1$$

$$\text{联立: } \begin{cases} y = \frac{1}{3}x + 1 \\ y = -x^2 - 2x + 3 \end{cases}, \text{ 解得: } x_1 = \frac{2}{3}, x_2 = -3 \text{ (舍)}$$



(第 24 题图 1)

$\therefore P$ 点坐标为 $(\frac{2}{3}, \frac{11}{9})$ 5 分

(II) 当 P 点在第四象限时, (如图 2)

同理可得: 直线 AF 的解析式为 $y = -\frac{1}{3}x - 1$

$$\text{联立: } \begin{cases} y = -\frac{1}{3}x - 1 \\ y = -x^2 - 2x + 3 \end{cases},$$

解得: $x_1 = \frac{4}{3}, x_2 = -3$ (舍)

$\therefore P$ 点坐标为 $(\frac{4}{3}, -\frac{13}{9})$

综上所述: P 点坐标为 $(\frac{2}{3}, \frac{11}{9})$ 或 $(\frac{4}{3}, -\frac{13}{9})$ 7 分

(2) 设 $y_{BE} = k_1x + c$ ($k_1 \neq 0$) $y_{AC} = k_2x + c$ ($k_2 \neq 0$)

$\therefore k_1x + c = ax^2 - 2x + c$, 解得: $x_1 = 0, x_2 = \frac{k_1+2}{a}$

$\therefore x_B = \frac{k_1+2}{a}$ 8 分

同理: $x_A = \frac{k_2+2}{a}$,

$\therefore AB = \frac{k_1-k_2}{a}$ 9 分

又由于抛物线的对称轴为直线 $x = -\frac{-2}{2a} = \frac{1}{a}$

$\therefore y_E = k_1 \cdot \frac{1}{a} + c; y_F = k_2 \cdot \frac{1}{a} + c$ 10 分

$\therefore EF = (k_1 - k_2) \cdot \frac{1}{a}$ 11 分

$\therefore \frac{EF}{AB} = 1$ 12 分

