

五、(本大题共 2 小题，每小题 10 分，满分 20 分)

19. 解: $\because a = \sqrt{7} + 2, b = \sqrt{7} - 2,$

$$\therefore a + b = \sqrt{7} + 2 + \sqrt{7} - 2 = 2\sqrt{7}, \quad a - b = (\sqrt{7} + 2) - (\sqrt{7} - 2) = 4,$$

$$(1) \quad a^2 - 2ab + b^2 = (a - b)^2 = 4^2 = 16; \quad \dots\dots (5 \text{ 分})$$

$$(2) \quad a^2 - b^2 = (a + b)(a - b) = 2\sqrt{7} \times 4 = 8\sqrt{7}. \quad \dots\dots (10 \text{ 分})$$

20. 解:

(1) 连接 AP ,

$$\because PE \perp AB, PF \perp AC, BD \perp AC, \quad S_{\triangle ABC} = S_{\triangle ABP} + S_{\triangle ACP},$$

$$\text{即 } \frac{1}{2} AC \cdot BD = \frac{1}{2} AB \cdot PE + \frac{1}{2} AC \cdot PF,$$

$$\because AB = AC, \therefore BD = PE + PF; \quad \dots\dots (5 \text{ 分})$$

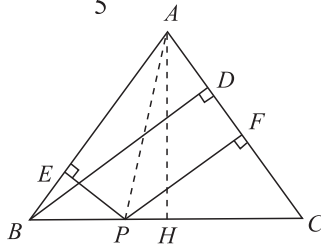
(2) 作 $AH \perp BC$,

$$\because \triangle ABC \text{ 是等腰三角形}, \therefore BH = \frac{1}{2} BC = 4,$$

$$\text{在 } \triangle AHB \text{ 中}, AH = \sqrt{AB^2 - BH^2} = \sqrt{5^2 - 4^2} = 3,$$

$$\because S_{\triangle ABC} = \frac{1}{2} AC \times BD = \frac{1}{2} BC \times AH, \therefore BD = \frac{BC \times AH}{AC} = \frac{8 \times 3}{5} = \frac{24}{5},$$

$$\therefore \text{由 (1) 可得 } PE + PF = BD = \frac{24}{5}. \quad \dots\dots (10 \text{ 分})$$



第 20 题图

六、(本题满分 12 分)

21. 解:

$$(1) \text{ 根据题意得 } \Delta = (\sqrt{2}c)^2 - 4ab = 2c^2 - 4ab,$$

$$\because a^2 + b^2 = c^2, \therefore 2c^2 - 4ab = 2(a^2 + b^2) - 4ab = 2(a - b)^2 \geq 0,$$

$$\text{即 } \Delta \geq 0, \therefore \text{勾系一元二次方程 } ax^2 + \sqrt{2}cx + b = 0 \text{ 必有实数根};$$

$$(2) \text{ 当 } x = -1 \text{ 时, 有 } a - \sqrt{2}c + b = 0, \text{ 即 } a + b = \sqrt{2}c,$$

$$\because 2a + 2b + \sqrt{2}c = 12\sqrt{2}, \text{ 即 } 2(a + b) + \sqrt{2}c = 12\sqrt{2},$$

$$\therefore 3\sqrt{2}c = 12\sqrt{2}, \therefore c = 4, \therefore a^2 + b^2 = c^2 = 16, \quad a + b = 4\sqrt{2},$$

$$\because (a + b)^2 = a^2 + b^2 + 2ab, \therefore ab = 8. \quad \dots\dots (12 \text{ 分})$$

七、(本题满分 12 分)

22. 解:

$$(1) \quad 45 + \frac{260 - 240}{10} \times 7.5 = 60; \quad \dots\dots (4 \text{ 分})$$

(2) 设售价每吨为 x 元,

$$\text{根据题意得 } (x - 100) \left(45 + \frac{260 - x}{10} \times 7.5 \right) = 9000,$$

化简得 $x^2 - 420x + 44000 = 0$ ，解得 $x_1 = 200$ ， $x_2 = 220$ （舍去），
 \therefore 将售价定为 200 元时销量最大.

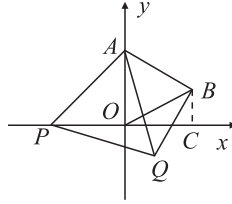
..... (12 分)

八、(本题满分 14 分)

23. 解:

- (1) 如图 1，过点 B 作 $BC \perp x$ 轴于点 C ，
 $\because \triangle AOB$ 为等边三角形，且 $OA = 4$ ， $\therefore \angle AOB = 60^\circ$ ， $OB = OA = 4$ ，
 $\therefore \angle BOC = 30^\circ$ ， $\therefore BC = \frac{1}{2} OB = 2$ ， $\therefore OC = 2\sqrt{3}$ ，
 \therefore 点 B 的坐标为 $B(2\sqrt{3}, 2)$;

..... (5 分)



第 23 题图

- (2) $\angle ABQ = 90^\circ$ ，始终不变. 理由如下:
 $\because \triangle APQ$ 、 $\triangle AOB$ 均为等边三角形， $\therefore AP = AQ$ ， $AO = AB$ ， $\angle PAQ = \angle OAB$ ，
 $\therefore \angle PAO = \angle QAB$ ， $\therefore \triangle APO \cong \triangle AQB(SAS)$ ， $\therefore \angle ABQ = \angle AOP = 90^\circ$ ； (9 分)
- (3) 如图 2，连接 OQ ，过点 Q 作 $QH \perp y$ 轴于点 H ，
 $\because AB \parallel OQ$ ， $\angle ABQ = 90^\circ$ ， $\angle ABO = \angle BAO = 60^\circ$ ，
 $\therefore \angle BQO = 90^\circ$ ， $\angle HOQ = \angle BOQ = 60^\circ$ ， $\therefore \angle OBQ = \angle OQH = 30^\circ$ ，
又 $\because OB = 4$ ， $\therefore OQ = \frac{1}{2} OB = 2$ ， $\therefore BQ = 2\sqrt{3}$ ， $OH = \frac{1}{2} OQ = 1$ ，
 $\because \triangle APO \cong \triangle AQB$ ， $\therefore OP = BQ = 2\sqrt{3}$ ，
 $\therefore S_{\triangle OPQ} = \frac{1}{2} OP \cdot OH = \frac{1}{2} \times 2\sqrt{3} \times 1 = \sqrt{3}$.

..... (14 分)

