

2023 年湖北省宜昌市初中学业水平考试

数学试题参考答案

一、选择题（每题 3 分，计 33 分）

题号	1	2	3	4	5	6	7	8	9	10	11
答案	A	D	C	B	B	A	C	C	D	D	D

二、填空题（每题 3 分，计 12 分）

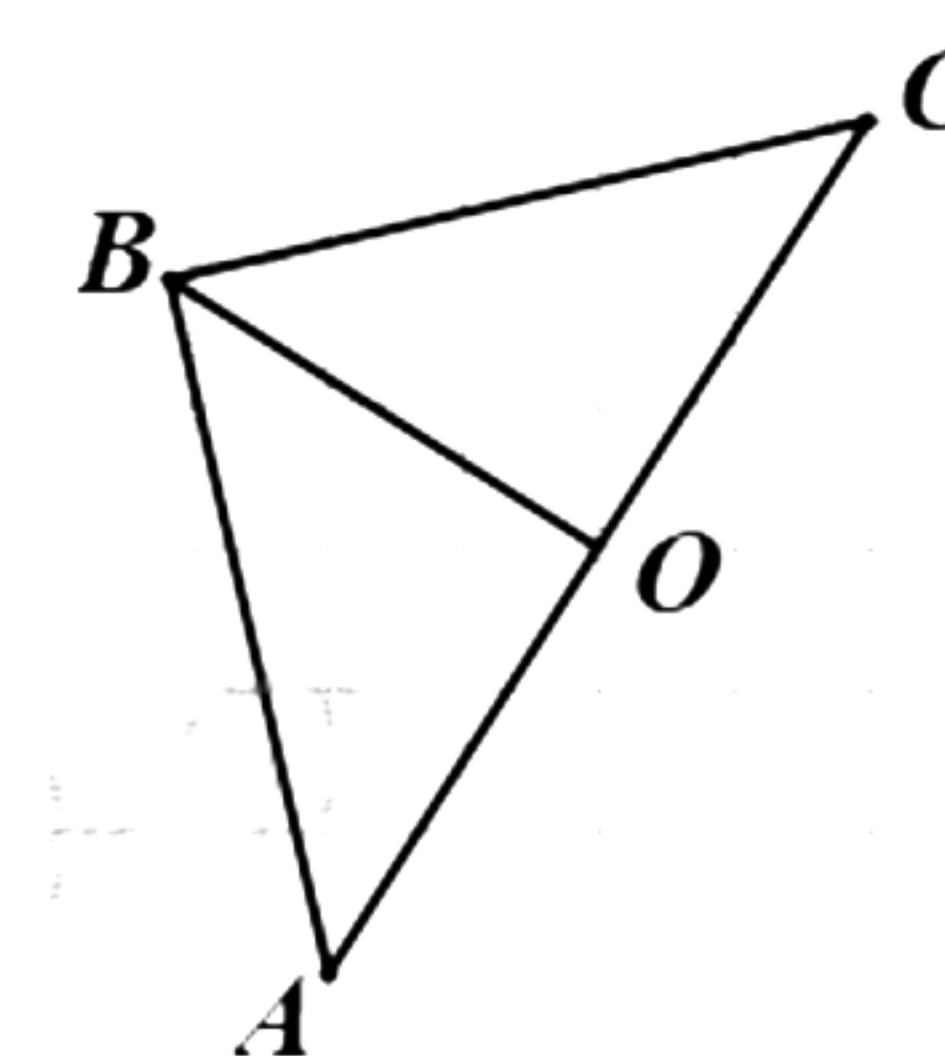
题号	12	13	14	15
答案	16	10	1	6

三、解答题（共 9 题，计 75 分）

16.（本题满分 6 分）解：原式 = $\frac{(a-2)^2}{(a+2)(a-2)} \times \frac{a(a+2)}{a-2} + 3 = a+3$

当 $a = \sqrt{3} - 3$ 时，原式 = $\sqrt{3} - 3 + 3 = \sqrt{3}$

17.（本题满分 6 分）解：如图，（1）在方格纸中画出线段 OA 绕点 O 顺时针旋转 90° 后得到的线段 OB ，连接 AB ；（2）再画出与 $\triangle AOB$ 关于直线 OB 对称的图形，点 A 的对称点是 C ；（3） $\angle OCB$ 的度数为 45° .



（第 17 题图）

18.（本题满分 7 分）解：（1）一次；

（2）设这个一次函数的解析式为 $y = kt + b (k \neq 0)$

\because 当 $t = 0$ 时， $y = 10$ ；当 $t = 10$ 时， $y = 30$

$$\therefore \begin{cases} 10 = b \\ 30 = 10k + b \end{cases} \quad \text{解得} \quad \begin{cases} k = 2 \\ b = 10 \end{cases}$$

$\therefore y$ 关于 t 的函数解析式为 $y = 2t + 10$

（3）当 $t = 110$ 时， $y = 2 \times 110 + 10 = 230$

答：当加热 110 s 时，油沸腾了，推算沸点的温度为 230°C .

19.（本题满分 7 分）解：（1）由题意可知， $PF = 330$

$$\because OP = OQ \approx 6400$$

$$\therefore OF = OP + PF = 330 + 6400 = 6730$$

$$\therefore \text{在 } Rt\triangle OFQ \text{ 中, } \cos \alpha = \frac{OQ}{OF} = \frac{6400}{6730} \approx 0.95$$

（2） $\because \cos \alpha \approx 0.95$ ， $\cos 18^\circ \approx 0.95$

$$\therefore \alpha = 18^\circ$$

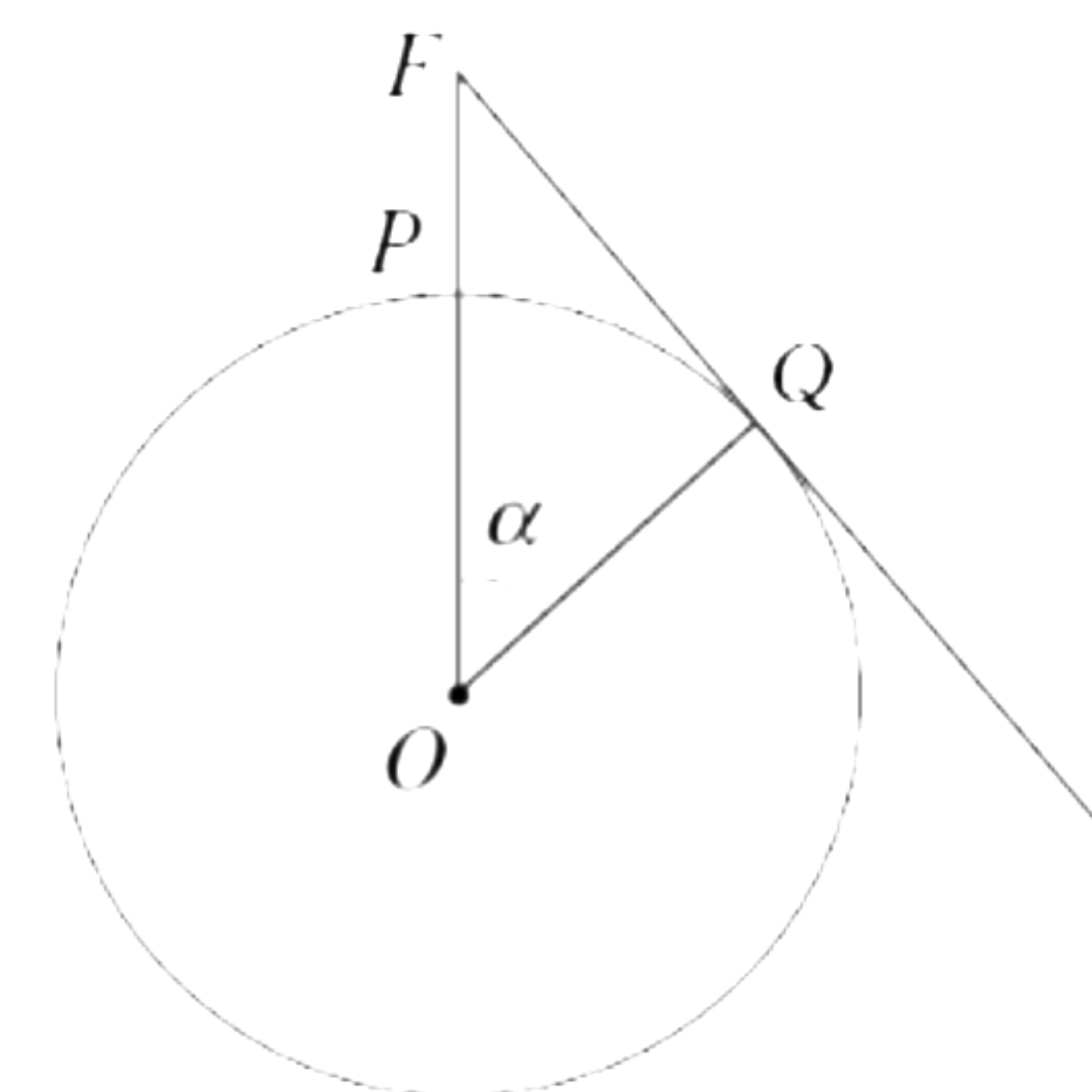
$$\therefore \widehat{PQ} \text{ 的长为 } l = \frac{18 \times \pi \times 6400}{180} = 640\pi$$

$$\approx 2009.6$$

$$\approx 2010$$

20.（本题满分 8 分）解：（1）80，32；（2） 72° ；（3）120；

$$\text{（4）} P(\text{小文、小明选择同一社团}) = \frac{4}{16} = \frac{1}{4}.$$



（第 19 题参考图）

小文选择 小明选择	文学	科幻	漫画	数理
文学	(文学, 文学)	(文学, 科幻)	(文学, 漫画)	(文学, 数理)
科幻	(科幻, 文学)	(科幻, 科幻)	(科幻, 漫画)	(科幻, 数理)
漫画	(漫画, 文学)	(漫画, 科幻)	(漫画, 漫画)	(漫画, 数理)
数理	(数理, 文学)	(数理, 科幻)	(数理, 漫画)	(数理, 数理)

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graph TD; Start[开始] --> Literature[文学]; Start --> SciFi[科幻]; Start --> Manga[漫画]; Start --> Math[数理]; Literature --> Lit_Lit[文学]; Literature --> Lit_SciFi[科幻]; Literature --> Lit_Manga[漫画]; Literature --> Lit_Math[数理]; SciFi --> SciFi_Lit[文学]; SciFi --> SciFi_SciFi[科幻]; SciFi --> SciFi_Manga[漫画]; SciFi --> SciFi_Math[数理]; Manga --> Manga_Lit[文学]; Manga --> Manga_SciFi[科幻]; Manga --> Manga_Manga[漫画]; Manga --> Manga_Math[数理]; Math --> Math_Lit[文学]; Math --> Math_SciFi[科幻]; Math --> Math_Manga[漫画]; Math --> Math_Math[数理];
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(2) 如图 1, AB 是 $\odot O$ 的直径

$$\because AB = 4, \quad PB = 3, \quad PA = 5$$

$$\therefore BC = \frac{12}{5}$$

$$\therefore AC = \frac{16}{5}$$

$$\therefore S_{\triangle ABC} = \frac{1}{2} \times \frac{16}{5} \times \frac{12}{5} = \frac{96}{25}$$

如图 2, $\angle ACB = \angle ABP = 90^\circ$

$$\therefore \angle APB = \angle ABC$$

$$\therefore \angle FEC = \angle ABC$$

$$\therefore \angle FEC = \angle APB$$

$$\therefore \angle AEC = \angle APG$$

$$\therefore \angle EAC = \angle PAG$$

$$\therefore \triangle EAC \sim \triangle PAG.$$

$$\therefore \frac{AC}{AG} = \frac{AE}{AP} = \frac{EC}{GP}$$

设 $EC = x$, $AE = 5x$

$$\therefore AP = 5$$

$$\therefore GP = 1$$

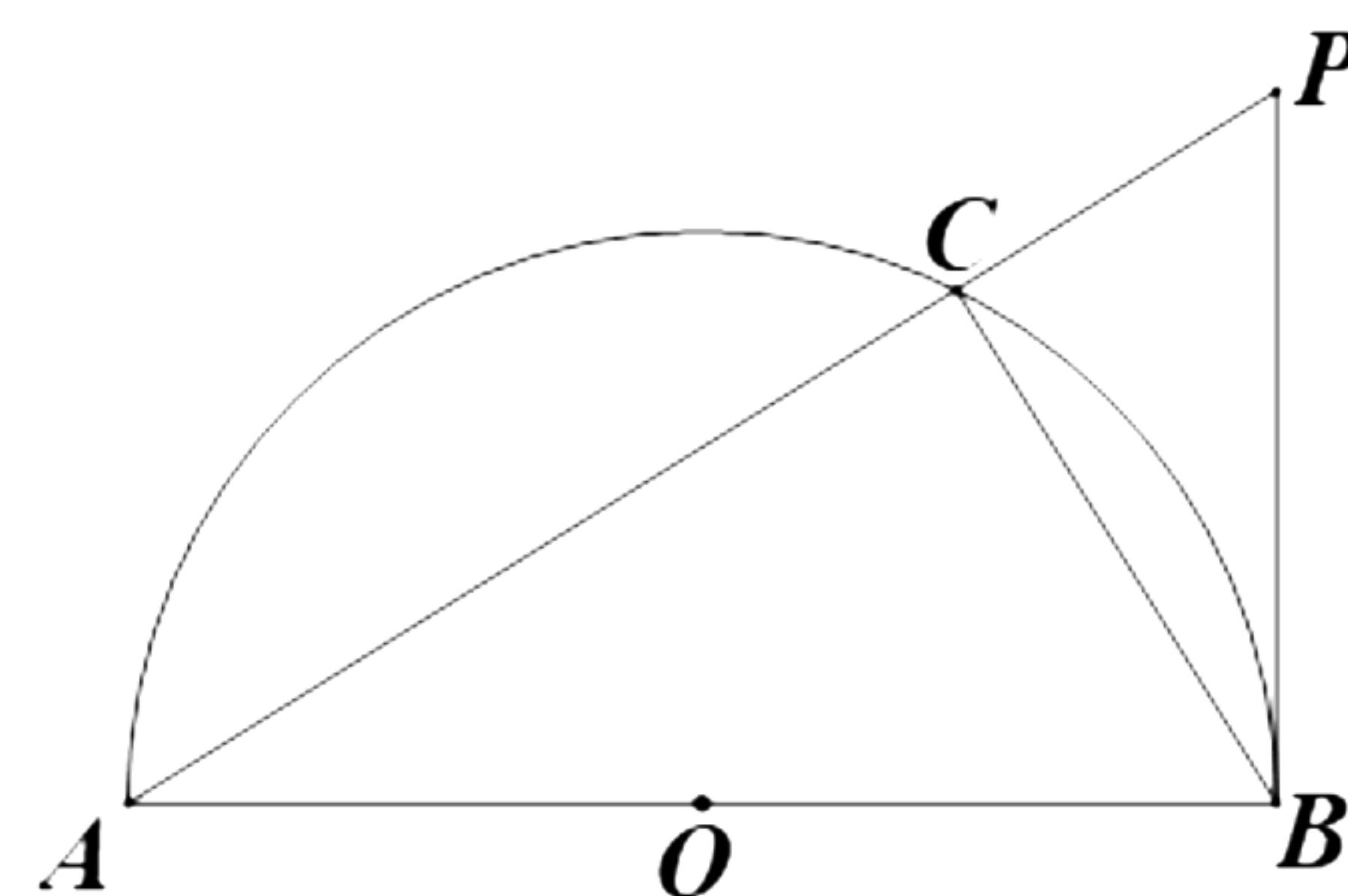
$$\therefore BG = BP + PG = 3 + 1 = 4$$

$$\therefore AB = BG$$

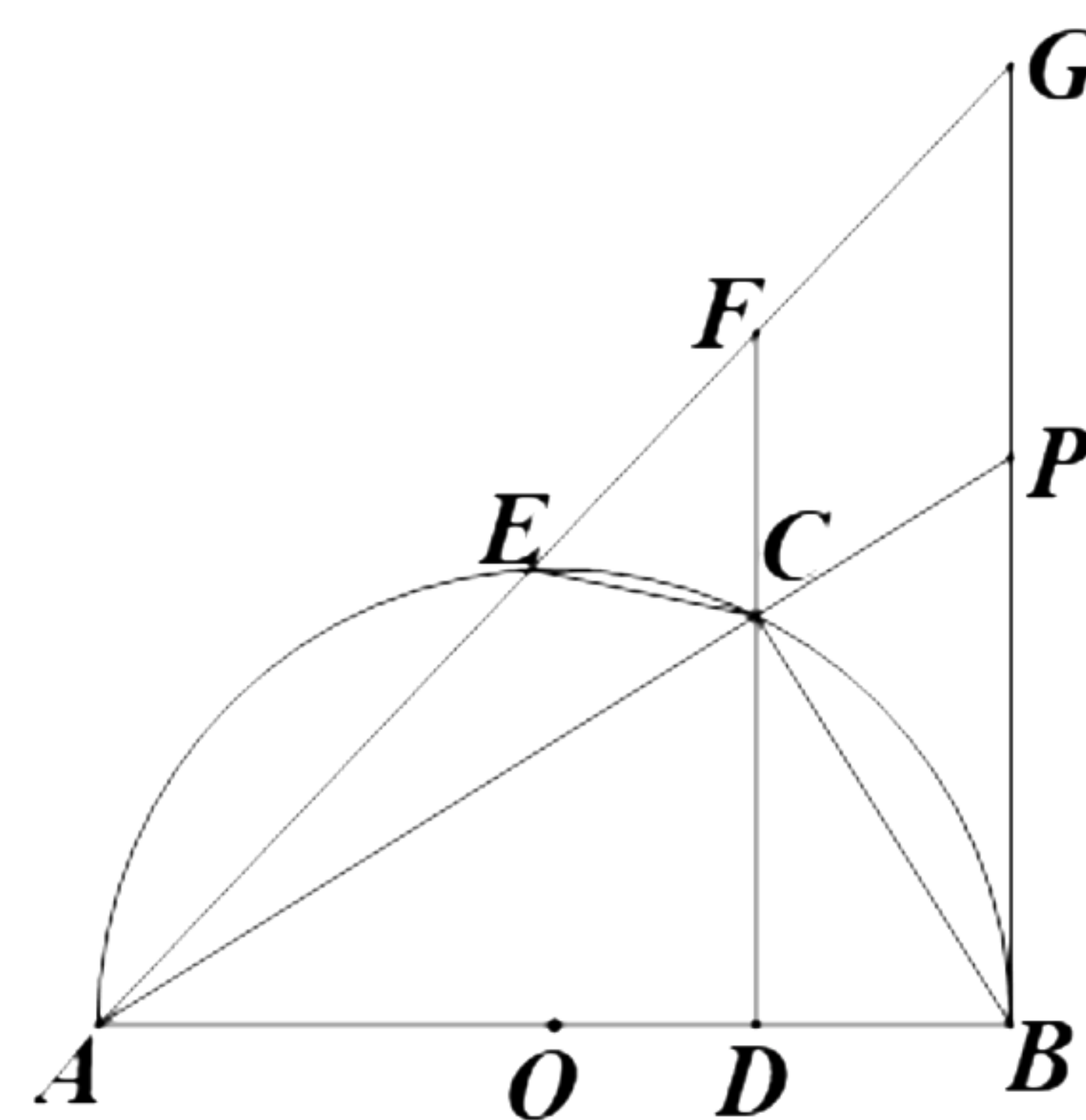
$\therefore \triangle ABG$ 是等腰直角三角形, $AG = 4\sqrt{2}$

$$\therefore AC = \frac{16}{5},$$

$$\therefore x = \frac{2\sqrt{2}}{5}$$



(第 21 题图 1)



(第 21 题图 2)

22. (本题满分 10 分) 解: (1) 设豆沙粽的单价为 x 元, 则肉粽的单价为 $2x$ 元

$$10x + 12 \times 2x = 136, \text{ 解得 } x = 4; \text{ 则 } 2x = 8;$$

所以豆沙粽的单价为 4 元, 肉粽的单价为 8 元;

(2) ① 设豆沙粽优惠后的单价为 a 元, 则肉粽优惠后的单价为 b 元

$$\begin{cases} 20a + 30b = 270 \\ 30a + 20b = 230 \end{cases} \text{ 解得 } \begin{cases} a = 3 \\ b = 7 \end{cases}$$

所以豆沙粽优惠后的单价为 3 元, 肉粽优惠后的单价为 7 元

$$\text{② } [3m + (40 - m) \times 7] \times (80 - 4m) + [3 \times (40 - m) + 7m] \times (4m + 8) = 17280$$

解得 $m = 19$ 或 $m = 10$

$$\because m < \frac{1}{2}(40 - m), \quad m < \frac{40}{3}$$

$$\therefore m = 10$$

23. (本题满分 11 分) 解: (1) 如图 1, 正方形 $ABCD$ 中, $CD = AD = 2$

$$\text{① } \because \angle ADC = \angle BAD = \angle FEC = 90^\circ$$

$$\therefore \angle AEF = \angle ECD$$

$$\therefore \triangle AEF \sim \triangle DCE$$

② 如图 2, 延长 DA , CF 交于点 G

作 $GH \perp CE$, 垂足为 H

$$\because \angle EDC = \angle EHG = 90^\circ \text{ 且 } \angle CED = \angle GEH$$

$$\therefore \triangle CED \sim \triangle GEH$$

$$\therefore \frac{GE}{CE} = \frac{GH}{CD} = \frac{EH}{ED}$$

$$\because CD = 2, \quad DE = 1$$

$$\therefore CE = \sqrt{5}$$

$$\text{方法一: 设 } EH = m, \quad GH = 2m, \quad EG = \sqrt{5}m$$

$$\because \text{在 Rt}\triangle CHG \text{ 中, } \tan \angle FCE = \frac{GH}{CH} = \frac{2m}{\sqrt{5} + m} = \frac{2}{3}$$

$$\therefore m = \frac{\sqrt{5}}{2}$$

$$\therefore EG = \sqrt{5}m = \frac{5}{2}$$

方法二: 在 $\text{Rt}\triangle GHE$ 中, 设 $GH = 2n$, $CH = 3n$

$$\therefore \frac{3n - \sqrt{5}}{1} = \frac{2n}{2} = \frac{GE}{\sqrt{5}}$$

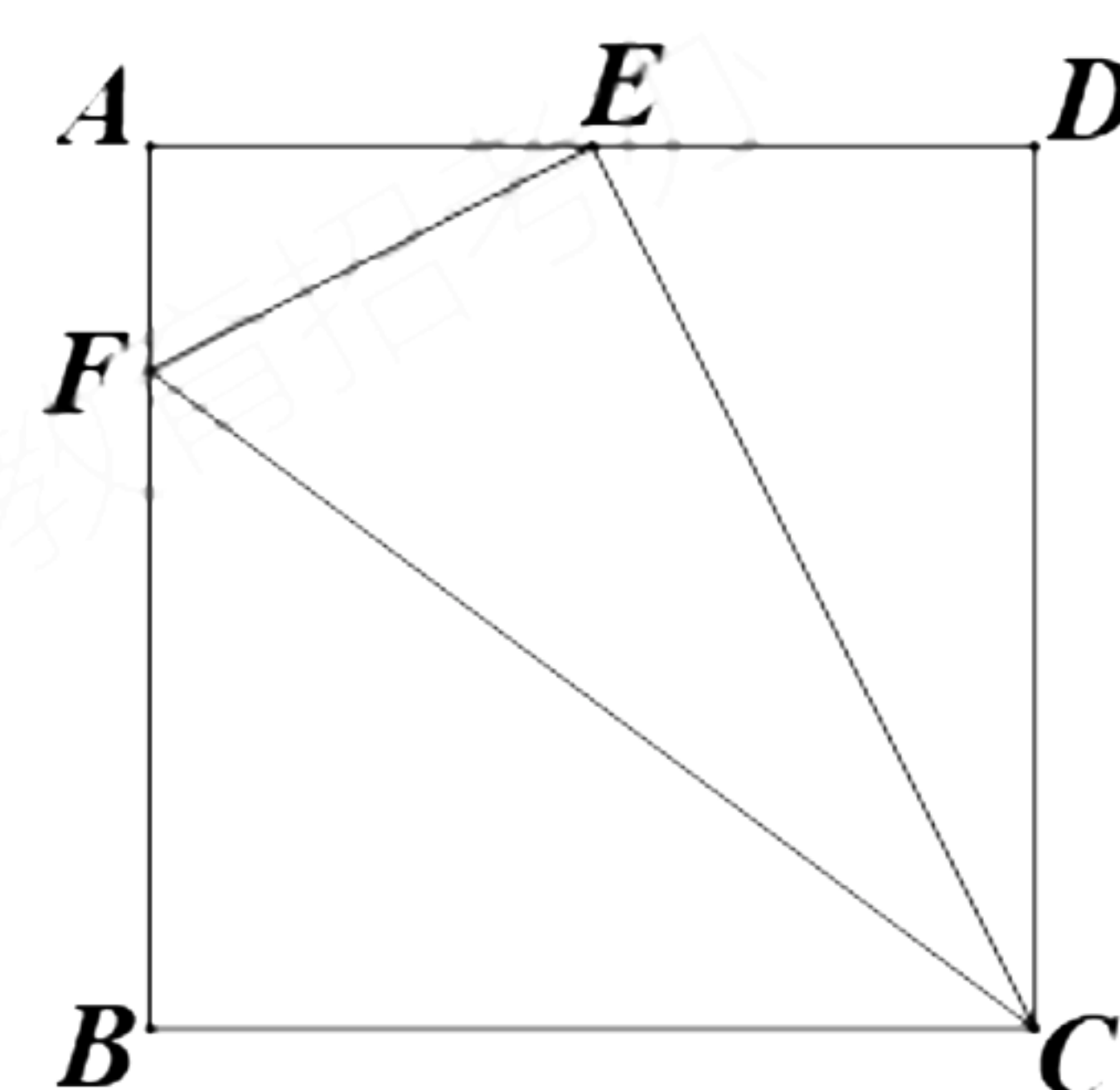
$$\therefore n = \frac{\sqrt{5}}{2}$$

$$\therefore GE = \sqrt{5}n = \frac{5}{2}$$

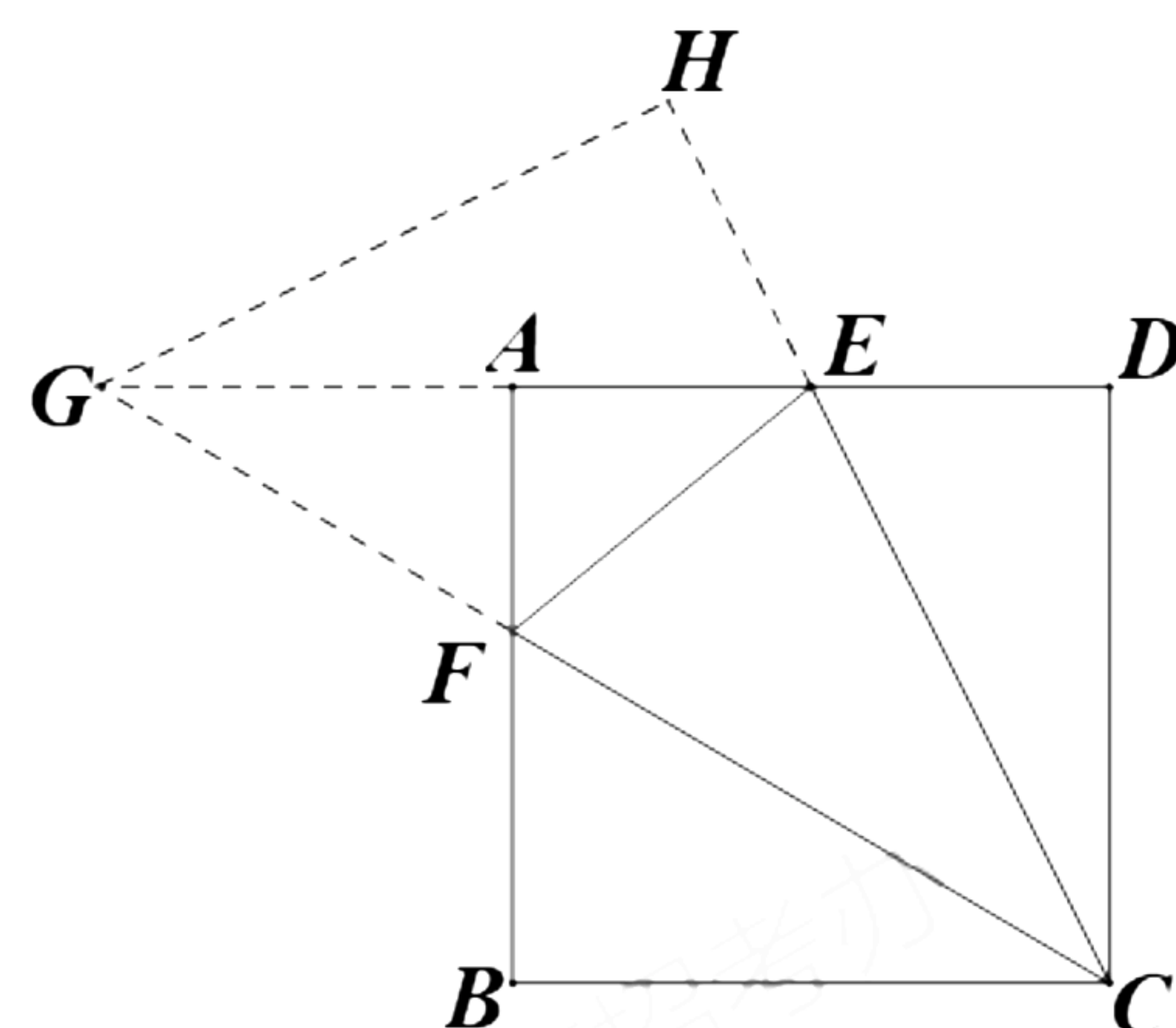
又 $\because \angle GAF = \angle GDC = 90^\circ$ 且 $\angle AGF = \angle DGC$

$$\therefore \triangle AGF \sim \triangle DGC$$

$$\therefore \frac{AG}{DG} = \frac{AF}{DC}$$



(第 23 题图 1)



(第 23 题图 2)

$$\therefore \frac{3}{2} : \frac{7}{2} = AF : 2$$

$$\therefore AF = \frac{6}{7}$$

(3) 如图 3, 延长 CE , 作 $GH \perp CE$, 垂足为 H

$$\because \angle EDC = \angle EHG = 90^\circ \text{ 且 } \angle CED = \angle GEH$$

$$\therefore \triangle CED \sim \triangle GEH$$

$$\text{设 } AD = CD = a, \quad GE = DE = t, \quad EH = x, \quad GH = y, \quad CE = n$$

$$\therefore \frac{x}{t} = \frac{y}{a} = \frac{t}{n}$$

$$\therefore x = \frac{t^2}{n}, \quad y = \frac{at}{n}$$

$$\text{在 Rt}\triangle CHG \text{ 中, } \sin \angle FCE = \frac{1}{3}$$

$$\therefore \tan \angle FCE = \frac{1}{2\sqrt{2}}$$

$$\therefore \frac{y}{x+n} = \frac{1}{2\sqrt{2}}$$

$$\therefore 2\sqrt{2}y = x + n$$

$$\therefore \frac{2\sqrt{2}at}{n} = \frac{t^2}{n} + n$$

$$\therefore 2\sqrt{2}at = t^2 + n^2$$

$$\because \text{在 Rt}\triangle CDE \text{ 中, } n^2 = t^2 + a^2$$

$$\therefore 2\sqrt{2}at = t^2 + t^2 + a^2$$

$$\therefore a^2 - 2\sqrt{2}at + 2t^2 = 0$$

$$\therefore (a - \sqrt{2}t)^2 = 0, \text{ 则 } a = \sqrt{2}t$$

$$\text{又 } \because \angle GAF = \angle GDC = 90^\circ \text{ 且 } \angle AGF = \angle DGC$$

$$\therefore \triangle AGF \sim \triangle DGC$$

$$\therefore \frac{AG}{DG} = \frac{AF}{DC}$$

$$\therefore \frac{AF}{a} = \frac{2t-a}{2t}$$

$$\therefore AF = \frac{a(2t-a)}{2t} = a - \frac{a^2}{2t} = a - \frac{2t^2}{2t} = a - t$$

$$\therefore AE = a - t$$

$$\therefore AE = AF$$

注: 其它正确解法, 均可得分

24. (本题满分 12 分) 解: (1) 等腰直角三角形

(2) 如图 1, $\triangle AOB$, $\triangle DOE$ 是等腰直角三角形

$$\therefore \angle AOB = \angle DOE = 90^\circ, \quad AO = OB, \quad OD = OE$$

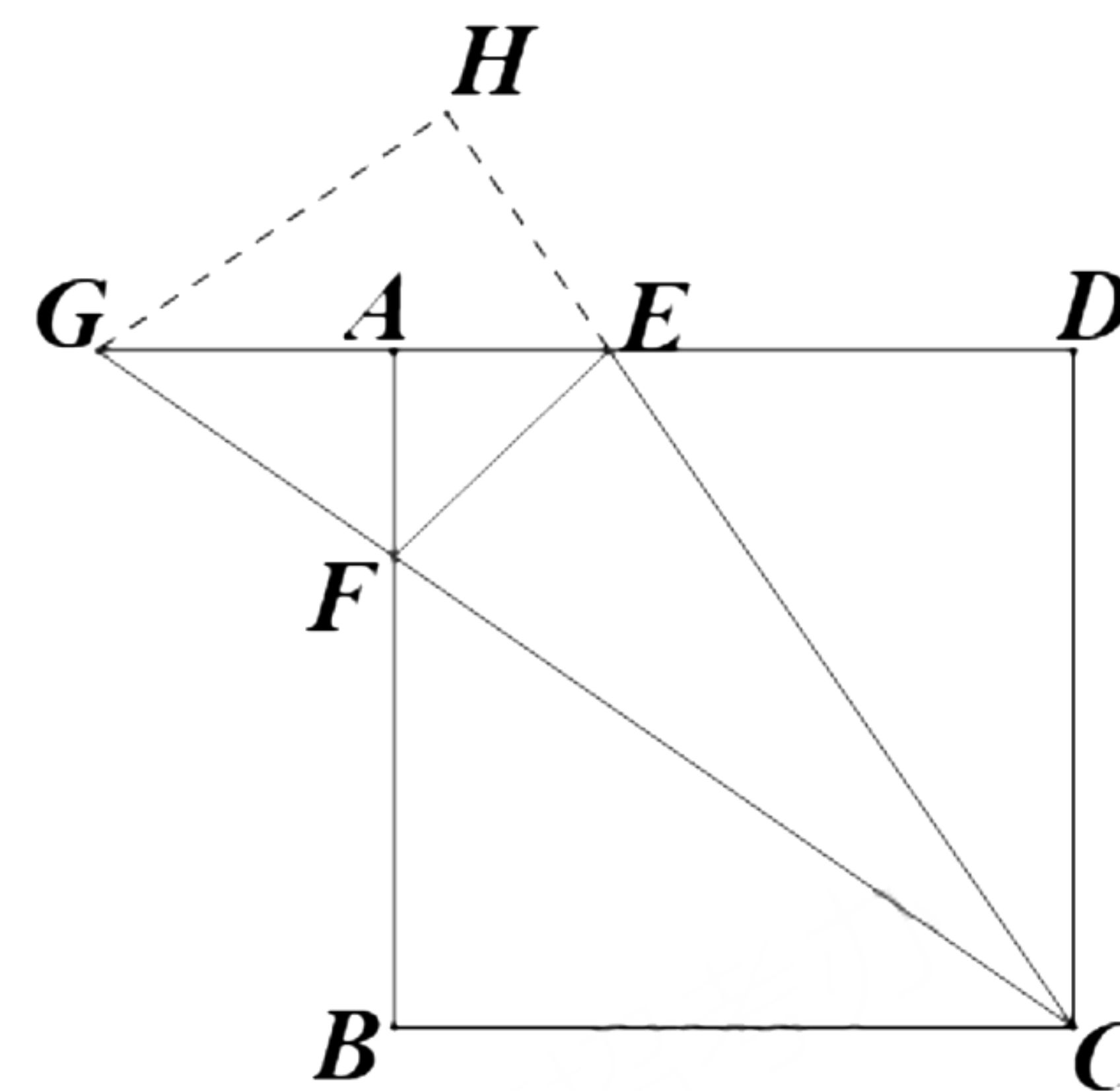
$$\therefore \angle AOE = \angle BOD$$

$$\therefore \triangle AOE \cong \triangle BOD \text{ (SAS)}$$

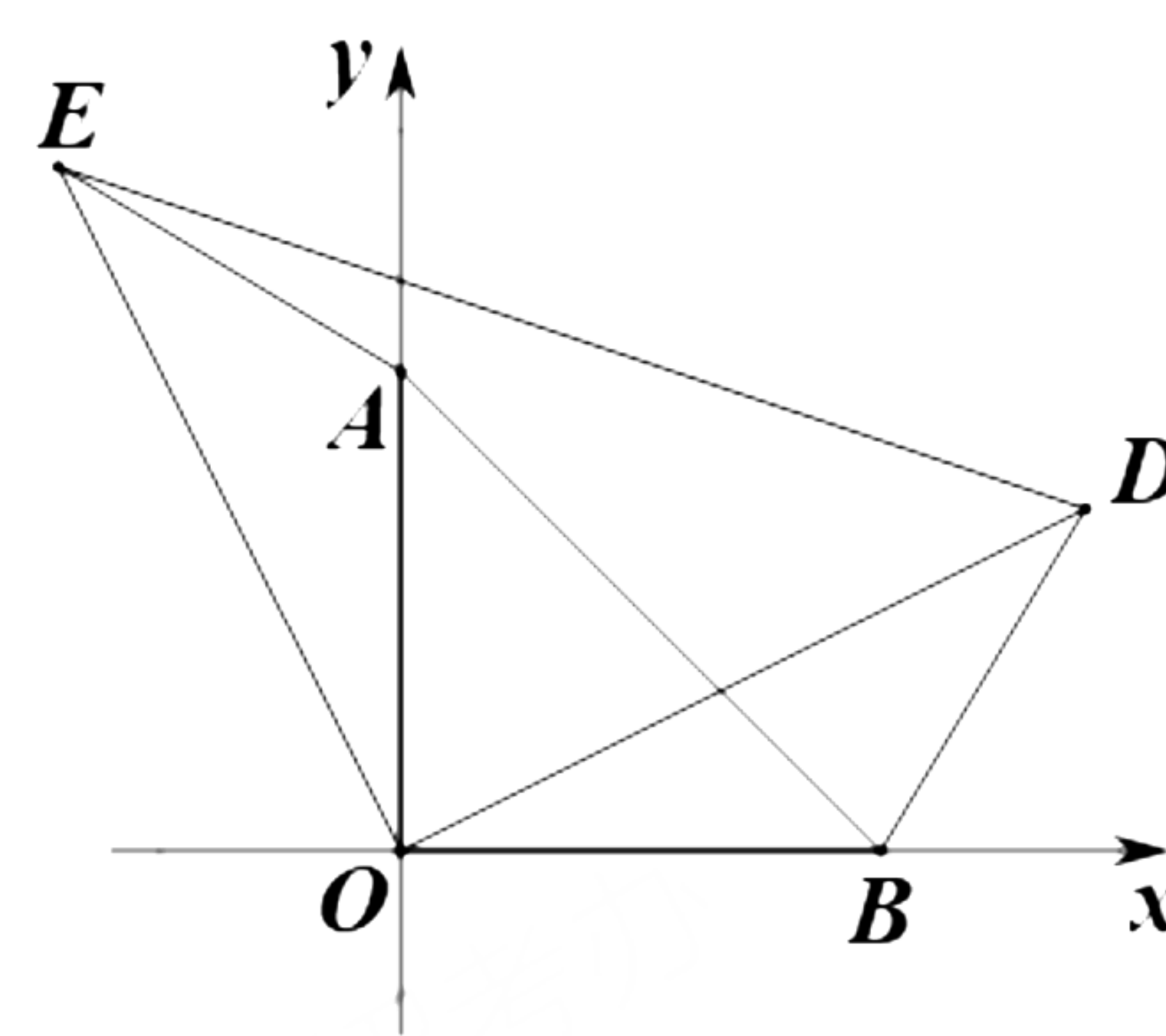
(3) ① $\because A(0, 2), C(t, 0)$

$$\therefore y_{AC} = -\frac{2}{t}x + 2$$

将 $C(t, 0), B(2, 0)$ 代入抛物线 $y_1 = ax^2 + bx - 4$



(第 23 题图 3)



(第 24 题图 1)

$$\therefore a = -\frac{2}{t}, \quad b = \frac{2}{t}(t+2)$$

$$\therefore y_1 = -\frac{2}{t}x^2 + \frac{2}{t}(t+2)x - 4$$

\therefore 直线 $y_{AC} = -\frac{2}{t}x + 2$ 与抛物线 $y_1 = -\frac{2}{t}x^2 + \frac{2}{t}(t+2)x - 4$ 有唯一交点

\therefore 联立解析式组成方程组解得 $x^2 - tx - 3x + 3t = 0$

$$\therefore (t+3)^2 - 4 \times 3t = (t-3)^2 = 0$$

$$\therefore t = 3$$

②如图 2, 抛物线 $y_1 = -\frac{2}{t}x^2 + \frac{2}{t}(t+2)x - 4$ 向左平移 2 个单位

$$\therefore \text{抛物线 } y_2 = -\frac{2}{t}x^2 + \frac{2}{t}(t-2)x \text{ 或 } y_2 = -\frac{2}{t}\left(x - \frac{t-2}{2}\right)^2 + \frac{(t-2)^2}{2t}$$

$$\therefore \text{顶点 } P\left(\frac{t-2}{2}, \frac{(t-2)^2}{2t}\right)$$

将顶点 $P\left(\frac{t-2}{2}, \frac{(t-2)^2}{2t}\right)$ 代入 $y_{AC} = -\frac{2}{t}x + 2$

$$\therefore t^2 - 6t = 0, \text{ 解得 } t_1 = 0, \quad t_2 = 6$$

由 $t > 2$, 得 $\therefore t = 6$

③过点 E 作 $EM \perp x$ 轴, 垂足为 M

过点 D 作 $DN \perp x$ 轴, 垂足为 N

$$\therefore \angle DOE = 90^\circ, \quad OD = OE$$

$$\therefore \triangle ODN \cong \triangle EOM \text{ (AAS)}$$

设 $EM = 2OM = 2m$

$$\therefore EM \perp x \text{ 轴}$$

$$\therefore OA \parallel EM$$

$$\therefore OC : CM = OA : EM$$

$$\therefore \frac{t}{t+m} = \frac{2}{2m}$$

$$\therefore m = \frac{t}{t-1}$$

$$\therefore D\left(\frac{2t}{t-1}, \frac{t}{t-1}\right)$$

\therefore 抛物线 y_2 再向下平移 $\frac{2}{(t-1)^2}$ 个单位, 得到抛物线 y_3

$$\therefore \text{抛物线 } y_3 = -\frac{2}{t}x^2 + \frac{2}{t}(t-2)x - \frac{2}{(t-1)^2}$$

$$\therefore D\left(\frac{2t}{t-1}, \frac{t}{t-1}\right) \text{ 代入抛物线 } y_3 = -\frac{2}{t}x^2 + \frac{2}{t}(t-2)x - \frac{2}{(t-1)^2}$$

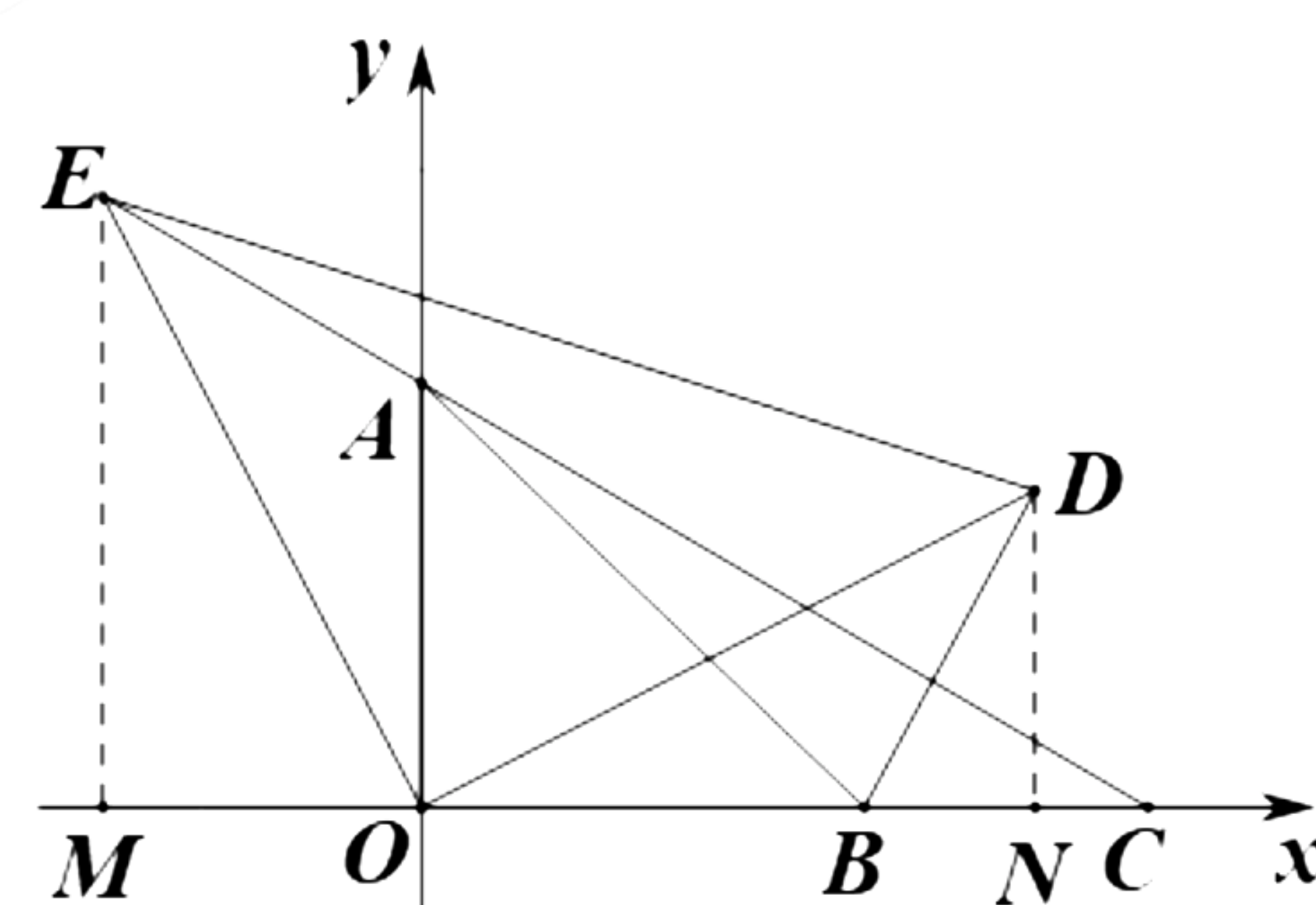
$$\therefore 3t^2 - 19t + 6 = 0$$

$$\text{解得 } t_1 = \frac{1}{3}, \quad t_2 = 6$$

由 $t > 2$, 得 $t = 6$

$$\therefore D\left(\frac{12}{5}, \frac{6}{5}\right)$$

注: 其它正确解法, 均可得分



(第 24 题图 2)